1. Derived copy of Mass, Energy, and the Theory of Relativity
2. modify p 566 table
3. Derived copy of The Brightness of Stars
4. Derived copy of Using Spectra to Measure Stellar Radius, Composition, and Motion
5. addition to p 678 new figs
6. Derived copy of The H–R Diagram and Cosmic Distances
7. two new distance exercises for Chaper 19

Derived copy of Mass, Energy, and the Theory of Relativity

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Explain how matter can be converted into energy
- Describe the particles that make up atoms
- Describe the nucleus of an atom
- Understand the nuclear forces that hold atoms together
- Trace the nuclear reactions in the solar interior

As we have seen, energy cannot be created or destroyed, but only converted from one form to another. One of the remarkable conclusions derived by Albert Einstein (see Albert Einstein) when he developed his theory of relativity is that matter can be considered a form of energy too and can be converted into energy. Furthermore, energy can also be converted into matter. This seemed to contradict what humans had learned over thousands of years by studying nature. Matter is something we can see and touch, whereas energy is something objects have when they do things like move or heat up. The idea that matter or energy can be converted into each other seemed as outrageous as saying you could accelerate a car by turning the bumper into more speed, or that you could create a bigger front seat by slowing down your car. That would be pretty difficult to believe; yet, the universe actually works somewhat like that.

Converting Matter into Energy

The remarkable equivalence between matter and energy is given in one of the most famous equations:

Equation:

$$E = mc^2$$

In this equation, *E* stands for energy, *m* stands for mass, and *c*, the constant that relates the two, is the speed of light (3×10^8 meters per second). Note that mass is a measure of the quantity of matter, so the significance of this equation is that matter can be converted into energy and energy can be converted into matter. Let’s compare this equation of converting matter and energy to some common conversion equations that have the same form:

Equation:

$$\text{inches} = \text{feet} \ \times \ 12, \text{or cents} = \text{dollars} \ \times \ 100$$

Just as each conversion formula allows you to calculate the conversion of one thing into another, when we convert matter into energy, we consider how much mass the matter has. The conversion factor in this case turns out not to be either 12 or 100, as in our examples, but another constant quantity: the speed of light squared. Note that matter does not have to travel at the speed of light (or the speed of light squared) for this conversion to occur. The factor of c^2 is just the number that Einstein showed must be used to relate mass and energy.

Notice that this formula does not tell us *how* to convert mass into energy, just as the formula for cents does not tell us where to exchange coins for a dollar bill. The formulas merely tell us what the equivalent values are if we succeed in making the conversion. When Einstein first derived his formula in 1905, no one had the faintest idea how to convert mass into energy in any practical way. Einstein himself tried to discourage speculation that the large-scale conversion of atomic mass into energy would be feasible in the near future. Today, as a result of developments in nuclear physics, we regularly convert mass into energy in power plants, nuclear weapons, and high-energy physics experiments in particle accelerators.

Because the speed of light squared (c^2) is a very large quantity, the conversion of even a small amount of mass results in a very large amount of energy. For example, the complete conversion of 1 gram of matter (about 1/28 ounce, or approximately 1 paperclip) would produce as much energy as the burning of 15,000 barrels of oil.

Scientists soon realized that the conversion of mass into energy is the source of the Sun’s heat and light. With Einstein’s equation of $E = mc^2$, we can calculate that the amount of energy radiated by the Sun could be produced by the complete conversion of about 4 million tons of matter into energy inside the Sun each second. Destroying 4 million tons per second sounds like a lot when compared to earthly things, but bear in mind that the Sun is a very big reservoir of matter. In fact, we will see that the Sun contains more than enough mass to destroy such huge amounts of matter and still continue shining at its present rate for billions of years.

But knowing all that still does not tell us *how* mass can be converted into energy. To understand the process that actually occurs in the Sun, we need to explore the structure of the atom a bit further.

Note:

Albert Einstein

For a large part of his life, Albert Einstein ([link]) was one of the most recognized celebrities of his day. Strangers stopped him on the street, and people all over the world asked him for endorsements, advice, and assistance. In fact, when Einstein and the great film star Charlie Chaplin met in California, they found they shared similar feelings about the loss of privacy that came with fame. Einstein’s name was a household word despite the fact that most people did not understand the ideas that had made him famous.

Einstein was born in 1879 in Ulm, Germany. Legend has it that he did not do well in school (even in arithmetic), and thousands of students have since attempted to justify a bad grade by referring to this story. Alas, like many legends, this one is not true. Records indicate that although he tended to rebel against the authoritarian teaching style in vogue in Germany at that time, Einstein was a good student.

After graduating from the Federal Polytechnic Institute in Zurich, Switzerland, Einstein at first had trouble getting a job (even as a high school teacher), but he eventually became an examiner in the Swiss Patent Office. Working in his spare time, without the benefit of a university environment but using his superb physical intuition, he wrote four papers in 1905 that would ultimately transform the way physicists looked at the world. One of these, which earned Einstein the Nobel Prize in 1921, set part of the foundation of *quantum mechanics*—the rich, puzzling, and remarkable theory of the subatomic realm. But his most important paper presented the *special theory of relativity*, a reexamination of space, time, and motion that added a whole new level of sophistication to our understanding of those concepts. The famed equation $E = mc^2$ was actually a relatively minor part of this theory, added in a later paper.

In 1916, Einstein published his *general theory of relativity*, which was, among other things, a fundamentally new description of gravity (see Black Holes and Curved Spacetime). When this theory was confirmed by measurements of the “bending of starlight” during a 1919 eclipse (*The New York Times* headline read, “Lights All Askew in the Heavens”), Einstein became world famous.

In 1933, to escape Nazi persecution, Einstein left his professorship in Berlin and settled in the United States at the newly created Institute for Advanced Studies at Princeton. He remained there until his death in 1955, writing, lecturing, and espousing a variety of intellectual and political causes. For example, he agreed to sign a letter written by Leo Szilard and other scientists in 1939, alerting President Roosevelt to the dangers of allowing Nazi Germany to develop the atomic bomb first. And in 1952, Einstein was offered the second presidency of Israel. In declining the position, he said, “I know a little about nature and hardly anything about men.”

Albert Einstein (1879–1955).

This portrait of Einstein was taken in 1912. (credit: modification of work by J. F. Langhans)

Elementary Particles

The fundamental components of atoms are the proton, neutron, and electron (see The Structure of the Atom).

Protons, neutrons, and electrons are by no means all the particles that exist. First, for each kind of particle, there is a corresponding but opposite *antiparticle*. If the particle carries a charge, its antiparticle has the opposite charge. The antielectron is the *positron*, which has the same mass as the electron but is positively charged. Similarly, the antiproton has a negative charge. The remarkable thing about such *antimatter* is that when a particle comes into contact with its antiparticle, the original particles are annihilated, and substantial amounts of energy in the form of photons are produced.

Since our world is made exclusively of ordinary particles of matter, antimatter cannot survive for very long. But individual antiparticles are found in cosmic rays (particles that arrive at the top of Earth's atmosphere from space) and can be created in particle accelerators. And, as we will see in a moment, antimatter is created in the core of the Sun and other stars.

Science fiction fans may be familiar with antimatter from the *Star Trek* television series and films. The Starship Enterprise is propelled by the careful combining of matter and antimatter in the ship's engine room. According to $E = mc^2$, the annihilation of matter and antimatter can produce a huge amount of energy, but keeping the antimatter fuel from touching the ship before it is needed must be a big problem. No wonder Scotty, the chief engineer in the original TV show, always looked worried!

In 1933, physicist Wolfgang Pauli ([link]) suggested that there might be another type of elementary particle. Energy seemed to disappear when certain types of nuclear reactions took place, violating the law of conservation of energy. Pauli was reluctant to accept the idea that one of the basic laws of physics was wrong, and he suggested a "desperate remedy." Perhaps a so-far-undetected particle, which was given the name **neutrino** ("little neutral one"), carried away the "missing" energy. He suggested that neutrinos were particles with zero mass, and that like photons, they moved with the speed of light.

Wolfgang Pauli in 1945.

Pauli is considered the "father" of the neutrino, having conceived of it in 1933.

The elusive neutrino was not detected until 1956. The reason it was so hard to find is that neutrinos interact very weakly with other matter and therefore are very difficult to detect. Earth is more transparent to a neutrino than the thinnest and cleanest pane of glass is to a photon of light. In fact, most neutrinos can pass completely through a star or planet without being absorbed. As we shall see, this behavior of neutrinos makes them a very important tool for studying the Sun. Since Pauli's prediction, scientists have learned a lot more about the neutrino. We now know that there are three different types of neutrinos, and in 1998, neutrinos were discovered to have a tiny amount of mass. Indeed, it is so tiny that electrons are at least 500,000 times more massive. Ongoing research is focused on determining the mass of neutrinos more precisely, and it may still turn out that one of the three types is massless. We will return to the subject of neutrinos later in this chapter.

Some of the properties of the proton, electron, neutron, and neutrino are summarized in [link]. (Other subatomic particles have been produced by experiments with particle accelerators, but they do not play a role in the generation of solar energy.)

Particle	Mass (kg)	Mass MeV/c ^{2}	Charge
Proton	1.67265×10^{-27}	938.272	+1
Neutron	1.67495×10^{-27}	939.566	0
Electron	9.11×10^{-31}	0.511	-1
Neutrino	$< 2 \times 10^{-36}$ (uncertain)	$< 1 \times 10^{-6}$	0

The Atomic Nucleus

The nucleus of an atom is not just a loose collection of elementary particles. Inside the nucleus, particles are held together by a very powerful force called the strong nuclear force. This is short-range force, only capable of acting over distances about the size of the atomic nucleus. A quick thought experiment shows how important this force is. Take a look at your finger and consider the atoms composing it. Among them is carbon, one of the basic elements of life. Focus your imagination on the nucleus of one of your carbon atoms. It contains six protons, which have a positive charge, and six neutrons, which are neutral. Thus, the nucleus has a net charge of six positives. If only the electrical force were acting, the protons in this and every carbon atom would find each other very repulsive and fly apart.

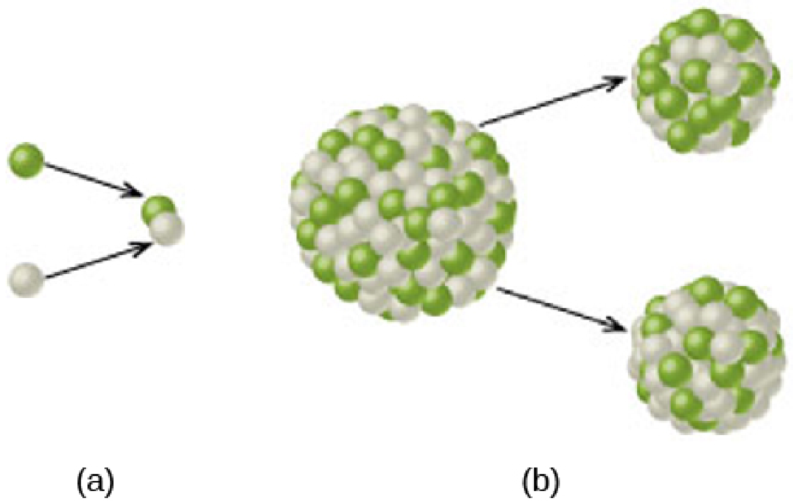
The strong nuclear force is an attractive force, stronger than the electrical force, and it keeps the particles of the nucleus tightly bound together. We saw earlier that if under the force of gravity a star “shrinks”—bringing its atoms closer together—gravitational energy is released. In the same way, if particles come together under the strong nuclear force and unite to form an atomic nucleus, some of the nuclear energy is released. The energy given up in such a process is called the *binding energy* of the nucleus.

When such binding energy is released, the resulting nucleus has slightly less mass than the sum of the masses of the particles that came together to form it. In other words, the energy comes from the loss of mass. This slight deficit in mass is only a small fraction of the mass of one proton. But because each bit of lost mass can provide a lot of energy (remember, $E = mc^2$), this nuclear energy release can be quite substantial.

Measurements show that the binding energy is greatest for atoms with a mass near that of the iron nucleus (with a combined number of protons and neutrons equal to 56) and less for both the lighter and the heavier nuclei. Iron, therefore, is the most stable element: since it gives up the most energy when it forms, it would require the most energy to break it back down into its component particles.

What this means is that, in general, when light atomic nuclei come together to form a heavier one (up to iron), mass is lost and energy is released. This joining together of atomic nuclei is called nuclear **fusion**.

Energy can also be produced by breaking up heavy atomic nuclei into lighter ones (down to iron); this process is called nuclear **fission**. Nuclear fission was the process we learned to use first—in atomic bombs and in nuclear reactors used to generate electrical power—and it may therefore be more familiar to you. Fission also sometimes occurs spontaneously in some unstable nuclei through the process of natural radioactivity. But fission requires big, complex nuclei, whereas we know that the stars are made up predominantly of small, simple nuclei. So we must look to fusion first to explain the energy of the Sun and the stars ([link]).
Fusion and Fission.



(a) In fusion, light atomic nuclei join together to form a heavier nuclei, releasing energy in the process. (b) In fission, energy is produced by the breaking up of heavy, complex nuclei into lighter ones.

Nuclear Attraction versus Electrical Repulsion

So far, we seem to have a very attractive prescription for producing the energy emitted by the Sun: “roll” some nuclei together and join them via nuclear fusion. This will cause them to lose some of their mass, which then turns into energy. However, every nucleus, even simple hydrogen, has protons—and protons all have positive charges. Since like charges repel via the electrical force, the closer we get two nuclei to each other, the more they repel. It’s true that if we can get them within “striking distance” of the nuclear force, they will then come together with a much stronger attraction. But that striking distance is very tiny, about the size of a nucleus. How can we get nuclei close enough to participate in fusion?

The answer turns out to be heat—tremendous heat—which speeds the protons up enough to overcome the electrical forces that try to keep protons apart. Inside the Sun, as we saw, the most common element is hydrogen, whose nucleus contains only a single proton. Two protons can fuse only in regions where the temperature is greater than about 12 million K, and the speed of the protons average around 1000 kilometers per second or more. (In old-fashioned units, that’s over 2 million miles per hour!)

In our Sun, such extreme temperatures are reached only in the regions near its center, which has a temperature of 15 million K. Calculations show that nearly all of the Sun’s energy is generated within about 150,000 kilometers of its core, or within less than 10% of its total volume.

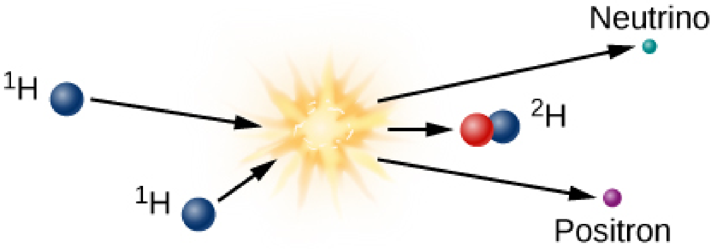
Even at these high temperatures, it is exceedingly difficult to force two protons to combine. On average, a proton will rebound from other protons in the Sun’s crowded core for about 14 billion years, at the rate of *100 million collisions per second*, before it fuses with a second proton. This is, however, only the *average* waiting time. Some of the enormous numbers of protons in the Sun’s inner region are “lucky” and take only a few collisions to achieve a fusion reaction: they are the protons responsible for producing the energy radiated by the Sun. Since the Sun is about 4.5 billion years old, most of its protons have not yet been involved in fusion reactions.

Nuclear Reactions in the Sun’s Interior

The Sun, then, taps the energy contained in the nuclei of atoms through nuclear fusion. Let’s look at what happens in more detail. Deep inside the Sun, a three-step process takes four hydrogen nuclei and fuses them together to form a single helium nucleus. The helium nucleus is slightly less massive than the four hydrogen nuclei that combine to form it, and that mass is converted into energy.

The initial step required to form one helium nucleus from four hydrogen nuclei is shown in [link]. At the high temperatures inside the Sun’s core, two protons combine to make a *deuterium* nucleus, which is an isotope (or version) of hydrogen that contains one proton and one neutron. In effect, one of the original protons has been converted into a neutron in the fusion reaction. Electric charge has to be conserved in nuclear reactions, and it is conserved in this one. A **positron** (antimatter electron) emerges from the reaction and carries away the positive charge originally associated with one of the protons.

Proton-Proton Chain, Step 1.



This is the first step in the process of fusing hydrogen into helium in the Sun. High temperatures are required because this reaction starts with two hydrogen nuclei, which are protons (shown in blue at left) that must overcome electrical repulsion to combine, forming a hydrogen nucleus with a proton and a neutron (shown in red). Note that

hydrogen containing one proton and one
neutron is given its own name: deuterium.
Also produced in this reaction are a positron,
which is an antielectron, and an elusive
particle named the neutrino.

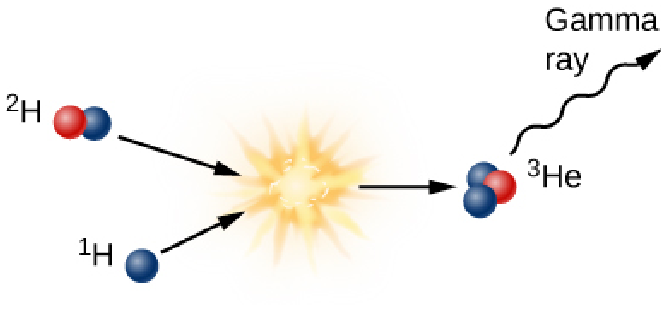
Since it is antimatter, this positron will instantly collide with a nearby electron, and both will be annihilated, producing electromagnetic energy in the form of gamma-ray photons. This gamma ray, which has been created in the center of the Sun, finds itself in a world crammed full of fast-moving nuclei and electrons. The gamma ray collides with particles of matter and transfers its energy to one of them. The particle later emits another gamma-ray photon, but often the emitted photon has a bit less energy than the one that was absorbed.

Such interactions happen to gamma rays again and again and again as they make their way slowly toward the outer layers of the Sun, until their energy becomes so reduced that they are no longer gamma rays but X-rays (recall what you learned in The Electromagnetic Spectrum). Later, as the photons lose still more energy through collisions in the crowded center of the Sun, they become ultraviolet photons.

By the time they reach the Sun’s surface, most of the photons have given up enough energy to be ordinary light—and they are the sunlight we see coming from our star. (To be precise, each gamma-ray photon is ultimately converted into many separate lower-energy photons of sunlight.) So, the sunlight given off by the Sun today had its origin as a gamma ray produced by nuclear reactions deep in the Sun’s core. The length of time that photons require to reach the surface depends on how far a photon on average travels between collisions, and the travel time depends on what model of the complicated solar interior we accept. Estimates are somewhat uncertain but indicate that the emission of energy from the surface of the Sun can lag its production in the interior by 100,000 years to as much as 1,000,000 years.

In addition to the positron, the fusion of two hydrogen atoms to form deuterium results in the emission of a neutrino. Because neutrinos interact so little with ordinary matter, those produced by fusion reactions near the center of the Sun travel directly to the Sun’s surface and then out into space, in all directions. Neutrinos move at nearly the speed of light, and they escape the Sun about two seconds after they are created.

Proton-Proton Chain, Step 2.

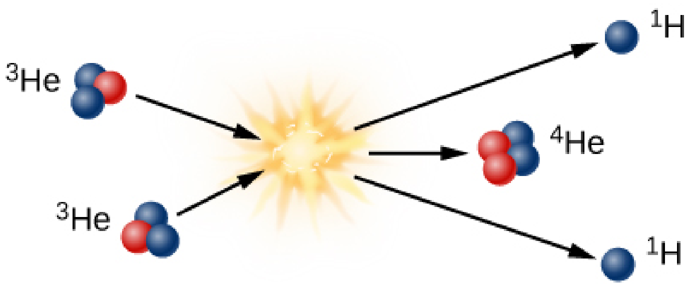


This is the second step of the proton-proton chain, the fusion reaction that converts hydrogen into helium in the Sun. This step combines one hydrogen nucleus, which is a proton (shown in blue), with the deuterium nucleus from the previous step (shown as a red and blue particle). The product of this is an isotope of helium with two protons (blue) and one neutron (red) and energy in the form of gamma-ray radiation.

The second step in forming helium from hydrogen is to add another proton to the deuterium nucleus to create a helium nucleus that contains two protons and one neutron ([link]). In the process, some mass is again lost and more gamma radiation is emitted. Such a nucleus is helium because an element is defined by its number of protons; any nucleus with two protons is called helium. But this form of helium, which we call helium-3 (and write in shorthand as ^3He) is not the isotope we see in the Sun's atmosphere or on Earth. That helium has two neutrons and two protons and hence is called helium-4 (^4He).

To get to helium-4 in the Sun, helium-3 must combine with another helium-3 in the third step of fusion (illustrated in [link]). Note that two energetic protons are left over from this step; each of them comes out of the reaction ready to collide with other protons and to start step 1 in the chain of reactions all over again.

Proton-Proton Chain, Step 3.



This is the third step in the fusion of hydrogen into helium in the Sun. Note that the two helium-3 nuclei from the second step (see [link]) must combine before the third step becomes possible. The two protons that come out of this step have the energy to collide with other protons in the Sun and start step one again.

Note:
These animations of proton-proton reactions show the steps required for fusion of hydrogen into helium in the Sun.

Note:
Visit the Tokamak Fusion Reactor at the General Atomics Lab in San Diego, CA, for an 8-minute tour.

The Proton-Proton Chain

The nuclear reactions in the Sun that we have been discussing can be described succinctly through the following nuclear formulas:
Equation:

$$\begin{aligned} &{}^1\text{H} + {}^1\text{H} \longrightarrow {}^2\text{H} + \text{e}^+ + v \\ &{}^2\text{H} + {}^1\text{H} \longrightarrow {}^3\text{He} + \gamma \\ &{}^3\text{He} + {}^3\text{He} \longrightarrow {}^4\text{He} + {}^1\text{H} + {}^1\text{H} \end{aligned}$$

Here, the superscripts indicate the total number of neutrons plus protons in the nucleus, e^+ is the positron, v is the neutrino, and γ indicates that gamma rays are emitted. Note that the third step requires two helium-3 nuclei to start; the first two steps must happen twice before the third step can occur.

Although, as we discussed, the first step in this chain of reactions is very difficult and generally takes a long time, the other steps happen more quickly. After the deuterium nucleus is formed, it survives an average of only about 6 seconds before being converted into ^{3}He. About a million years after that (on average), the ^{3}He nucleus will combine with another to form ^{4}He.

We can compute the amount of energy these reactions generate by calculating the difference in the initial and final masses. The masses of hydrogen and helium atoms in the units normally used by scientists are 1.007825 *u* and 4.00268 *u*, respectively. (The unit of mass, *u*, is defined to be 1/12 the mass of an atom of carbon, or approximately the mass of a proton.) Here, we include the mass of the entire atom, not just the nucleus, because electrons are involved as well. When hydrogen is converted into helium, two positrons are created (remember, the first step happens twice), and these are annihilated with two free electrons, adding to the energy produced.

Equation:

$$\begin{aligned} 4 \times 1.007825 &= 4.03130\,u \text{ (mass of initial hydrogen atoms)} \\ &- 4.00268\,u \text{ (mass of final helium atoms)} \\ &= 0.02862\,u \text{ (mass lost in the transformation)} \end{aligned}$$

The mass lost, 0.02862 *u*, is 0.71% of the mass of the initial hydrogen. Thus, if 1 kilogram of hydrogen is converted into helium, then the mass of the helium is only 0.9929 kilogram, and 0.0071 kilogram of material is converted into energy. The speed of light (c) is 3×10^8 meters per second, so the energy released by the conversion of just 1 kilogram of hydrogen into helium is:

Equation:

$$\begin{aligned} E &= mc^2 \\ E &= 0.0071\,\text{kg} \times \left(3 \times 10^8 \text{m/s}\right)^2 = 6.4 \times 10^{14}\,\text{J} \end{aligned}$$

This amount, the energy released when a single kilogram (2.2 pounds) of hydrogen undergoes fusion, would supply all of the electricity used in the United States for about 2 weeks.

To produce the Sun’s luminosity of 4×10^{26} watts, some 600 million tons of hydrogen must be converted into helium *each second*, of which about 4 million tons are converted from matter into energy. As large as these numbers are, the store of hydrogen (and thus of nuclear energy) in the Sun is still *more* enormous, and can last a long time—billions of years, in fact.

At the temperatures inside the stars with masses smaller than about 1.2 times the mass of our Sun (a category that includes the Sun itself), most of the energy is produced by the reactions we have just described, and this set of reactions is called the **proton-proton chain** (or sometimes, the p-p chain). In the proton-proton chain, protons collide directly with other protons to form helium nuclei.

In hotter stars, another set of reactions, called the carbon-nitrogen-oxygen (CNO) cycle, accomplishes the same net result. In the CNO cycle, carbon and hydrogen nuclei collide to initiate a series of reactions that form nitrogen, oxygen, and ultimately, helium. The nitrogen and oxygen nuclei do not survive but interact to form carbon again. Therefore, the outcome is the same as in the proton-proton chain: four hydrogen atoms disappear, and in their place, a single helium atom is created. The CNO cycle plays only a minor role in the Sun but is the main source of energy for stars with masses greater than about the mass of the Sun.

So you can see that we have solved the puzzle that so worried scientists at the end of the nineteenth century. The Sun can maintain its high temperature and energy output for billions of years through the fusion of the simplest element in the universe, hydrogen. Because most of the Sun (and the other stars) is made of hydrogen, it is an ideal “fuel” for powering a star. As will be discussed in the following chapters, we can define a star as a ball of gas capable of getting its core hot enough to initiate the fusion of hydrogen. There are balls of gas that lack the mass required to do this (Jupiter is a local example); like so many hopefuls in Hollywood, they will never be stars.

Note:

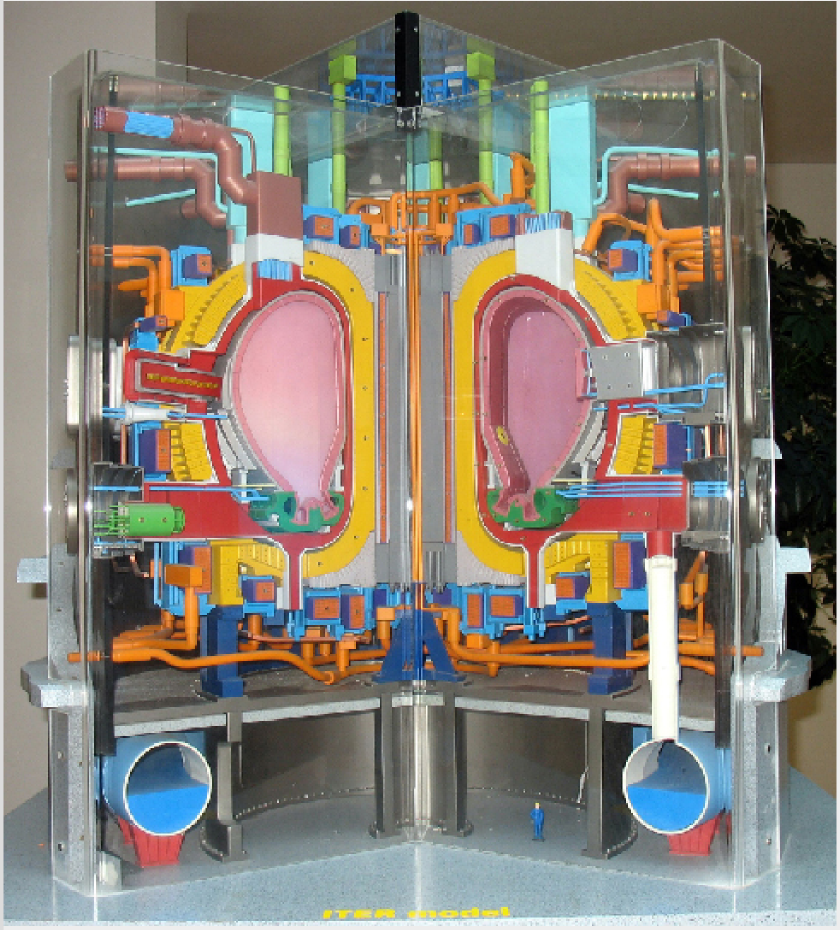
Fusion on Earth

Wouldn’t it be wonderful if we could duplicate the Sun’s energy mechanism in a controlled way on Earth? (We have already duplicated it in an uncontrolled way in hydrogen bombs, but we hope our storehouses of these will never be used.) Fusion energy would have many advantages: it would use hydrogen (or deuterium, which is heavy hydrogen) as fuel, and there is abundant hydrogen in Earth’s lakes and oceans. Water is much more evenly distributed around the world than oil or uranium, meaning that a few countries would no longer hold an energy advantage over the others. And unlike fission, which leaves dangerous byproducts, the nuclei that result from fusion are perfectly safe.

The problem is that, as we saw, it takes extremely high temperatures for nuclei to overcome their electrical repulsion and undergo fusion. When the first hydrogen bombs were exploded in tests in the 1950s, the “fuses” to get them hot enough were fission bombs. Interactions at such temperatures are difficult to sustain and control. To make fusion power on Earth, after all, we have to do what the Sun does: produce temperatures and pressures high enough to get hydrogen nuclei on intimate terms with one another. The European Union, the United States, South Korea, Japan, China, Russia, Switzerland, and India are collaborating on the International Thermonuclear Experimental Reactor (ITER), a project to demonstrate the feasibility of controlled fusion ([link]). The facility is being built in France. Construction will require over 10,000,000 components and 2000 workers for assembly. The date for the start of operations is yet to be determined.

ITER is based on the Tokamak design, in which a large doughnut-shaped container is surrounded by superconducting magnets to confine and control the hydrogen nuclei in a strong magnetic field. Previous fusion experiments have produced about 15 million watts of energy, but only for a second or two, and they have required 100 million watts to produce the conditions necessary to achieve fusion. The goal of ITER is to build the first fusion device capable of producing 500 million watts of fusion energy for up to 1000 seconds. The challenge is keeping the deuterium and tritium—which will participate in fusion reactions—hot enough and dense enough, for a long enough time to produce energy.

ITER Design.



The bright yellow areas in this model show where the superconducting magnets will circle the chamber within which fusion will take place. A huge magnet will keep the charged nuclei of heavy hydrogen confined. The goal is to produce 500 megawatts of energy. (credit: modification of work by Stephan Mosel)

Energy Conservation in Fusion Reactions

We can calculate the kinetic energy released in any one of the reactions shown in Figures 16.6-8. Let us do so using energy conservation in the units particle physicists generally use, electron volts (eV) introduced on p. 170 of Chapter 5. (If any later conversion to Joules is desired, it is given by $1\,\text{eV} = 1.602 \times 10^{-19}\text{J}$.) For Figure 16.6 – also the first line in the listing at the bottom of p. 569 – the sum of rest energies of the two protons on the left-hand-side must equal the sum of rest energies of the three particles on the right-hand-side plus the extra, kinetic energy of those three that goes into heating the Sun. (Strictly speaking, this is the kinetic energy in excess of the initial kinetic energy of the two protons.) Moving all of the rest energies to the left-hand-side gives that kinetic energy:

Equation:

$$E_{0\,{}^1H} + E_{0\,{}^1H} - E_{0\,{}^2H} - E_{0\,e^+} - E_{0\,\nu} = KE$$

We then use Einstein’s famous equation relating the particles' rest energies to their invariant masses, $E_0 = mc^2$. Notice that the masses in Table 16.1 are given in two sets of units, with the right-hand column most suited to this problem. For each of the two protons,

Equation:

$$E_{0\,{}^1H} = E_{0p} = m_p c^2 = \left(938.272\ \text{MeV}/\text{c}^2\right)c^2 = 938.272\,\text{MeV}.$$

The deuteron is composed of a proton and a neutron, but its mass is slightly smaller than the simple sum of those masses in Table 16.1.

Equation:

$$E_{0\,{}^2H} = E_{0d} = m_d c^2 = \left(1875.61339\ \text{MeV}/\text{c}^2\right)c^2 = 1875.61339\,\text{MeV}.$$

The positron mass is identical to that of the electron (though it has the opposite charge), so we have

Equation:

$$E_{0\,e^{+}} = E_{0e} = m_e c^2 = \left(0.511\ \text{MeV}\ /\text{c}^2\right)c^2 = 0.511\,\text{MeV}.$$

The neutrino mass is as yet unknown precisely but is about 1 eV or slightly less so we can neglect it in the present problem involving sums of about a million eV. Thus, the kinetic energy of the three resultant particles that goes into heating the Sun from this reaction is

Equation:

$$938.272\,\text{MeV} + 938.272\,\text{MeV} - 1875.613\,\text{MeV} - 0.511\,\text{MeV} = 0.42\,\text{MeV} = K\text{E}.$$

Key Concepts and Summary

Solar energy is produced by interactions of particles—that is, protons, neutrons, electrons, positrons, and neutrinos. Specifically, the source of the Sun’s energy is the fusion of hydrogen to form helium. The series of reactions required to convert hydrogen to helium is called the proton-proton chain. A helium atom is about 0.71% less massive than the four hydrogen atoms that combine to form it, and that lost mass is converted to energy (with the amount of energy given by the formula $E = mc^2$).

Glossary

fission	breaking up of heavier atomic nuclei into lighter ones
fusion	building up of heavier atomic nuclei from lighter ones
neutrino	fundamental particle that has no charge and a mass that is tiny relative to an electron; it rarely interacts with ordinary matter and comes in three different types
positron	particle with the same mass as an electron, but positively charged
proton-proton chain	series of thermonuclear reactions by which nuclei of hydrogen are built up into nuclei of helium

modify p 566 table
modify p 566 table to include masses expressed in MeV

Particle	Mass (kg)	Mass (MeV/c^2)	Charge
Proton	1.67265×10^{-27}	938.272	+1
Neutron	1.67495×10^{-27}	939.566	0
Electron	9.11×10^{-31}	0.511	-1
Neutrino	$< 2 \times 10^{-36}$ (uncertain)	$< 1 \times 10^{-6}$	0

Derived copy of The Brightness of Stars

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Explain the difference between luminosity and apparent brightness
- Understand how astronomers specify brightness with magnitudes

Luminosity

Perhaps the most important characteristic of a star is its **luminosity**—the total amount of energy at all wavelengths that it emits per second. Earlier, we saw that the Sun puts out a tremendous amount of energy every second. (And there are stars far more luminous than the Sun out there.) To make the comparison among stars easy, astronomers express the luminosity of other stars in terms of the Sun's luminosity. For example, the luminosity of Sirius is about 25 times that of the Sun. We use the symbol L_{Sun} to denote the Sun's luminosity; hence, that of Sirius can be written as 25 L_{Sun}. In a later chapter, we will see that if we can measure how much energy a star emits and we also know its mass, then we can calculate how long it can continue to shine before it exhausts its nuclear energy and begins to die.

Apparent Brightness

Astronomers are careful to distinguish between the luminosity of the star (the total energy output) and the amount of energy that happens to reach our eyes or a telescope on Earth. Stars are democratic in how they produce radiation; they emit the same amount of energy in every direction in space. Consequently, only a minuscule fraction of the energy given off by a star actually reaches an observer on Earth. We call the amount of a star's energy that reaches a given area (say, one square meter) each second here on Earth its **apparent brightness**. If you look at the night sky, you see a wide range of apparent brightnesses among the stars. Most stars, in fact, are so dim that you need a telescope to detect them.

If all stars were the same luminosity—if they were like standard bulbs with the same light output—we could use the difference in their apparent brightnesses to tell us something we very much want to know: how far away they are. Imagine you are in a big concert hall or ballroom that is dark except for a few dozen 25-watt bulbs placed in fixtures around the walls. Since they are all 25-watt bulbs, their luminosity (energy output) is the same. But from where you are standing in one corner, they do *not* have the same apparent brightness. Those close to you appear brighter (more of their light reaches your eye), whereas those far away appear dimmer (their light has spread out more before reaching you). In this way, you can tell which bulbs are closest to you. In the same way, if all the stars had the same luminosity, we could immediately infer that the brightest-appearing stars were close by and the dimmest-appearing ones were far away.

To pin down this idea more precisely, recall from the Radiation and Spectra chapter that we know exactly how light fades with increasing distance. The energy we receive is inversely proportional to the square of the distance. If, for example, we have two stars of the same luminosity and one is twice as far away as the other, it will look four times dimmer than the closer one. If it is three times farther away, it will look nine (three squared) times dimmer, and so forth.

Alas, the stars do not all have the same luminosity. (Actually, we are pretty glad about that because having many different types of stars makes the universe a much more interesting place.) But this means that if a star looks dim in the sky, we cannot tell whether it appears dim because it has a low luminosity but is relatively nearby, or because it has a high luminosity but is very far away. To measure the luminosities of stars, we must first compensate for the dimming effects of distance on light, and to do that, we must know how far away they are. Distance is among the most difficult of all astronomical measurements. We will return to how it is determined after we have learned more about the stars. For now, we will describe how astronomers specify the apparent brightness of stars.

The Magnitude Scale

The process of measuring the apparent brightness of stars is called *photometry* (from the Greek *photo* meaning “light” and *–metry* meaning “to measure”). As we saw Observing the Sky: The Birth of Astronomy, astronomical photometry began with Hipparchus. Around 150 B.C.E., he erected an observatory on the island of Rhodes in the Mediterranean. There he prepared a catalog of nearly 1000 stars that included not only their positions but also estimates of their apparent brightnesses.

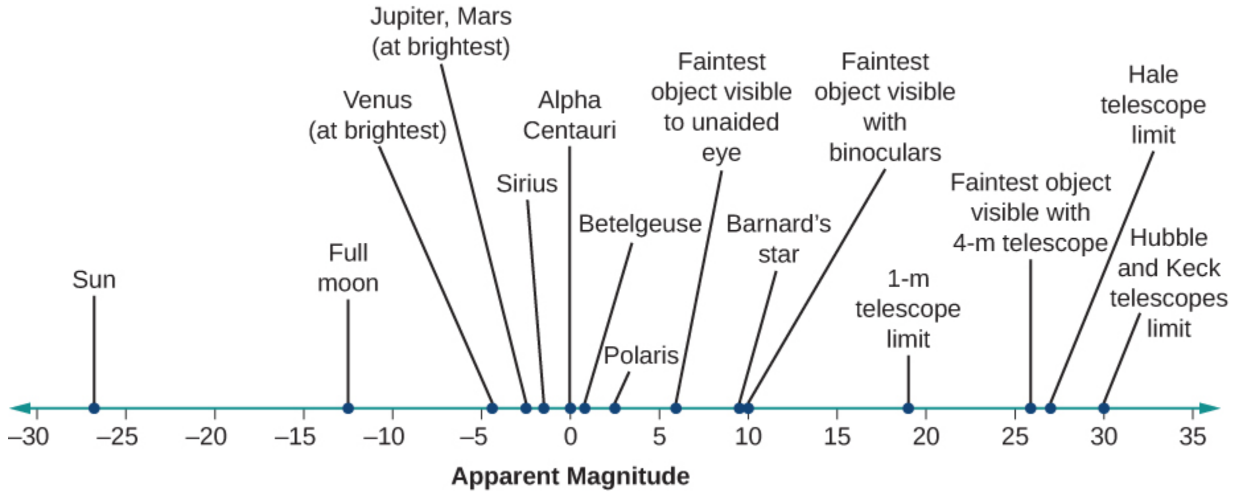
Hipparchus did not have a telescope or any instrument that could measure apparent brightness accurately, so he simply made estimates with his eyes. He sorted the stars into six brightness categories, each of which he called a **magnitude**. He referred to the brightest stars in his catalog as first-magnitudes stars, whereas those so faint he could barely see them were sixth-magnitude stars. During the nineteenth century, astronomers attempted to make the scale more precise by establishing exactly how much the apparent brightness of a sixth-magnitude star differs from that of a first-magnitude star. Measurements showed that we receive about 100 times more light from a first-magnitude star than from a sixth-magnitude star. Based on this measurement, astronomers then defined an accurate magnitude system in which a difference of five magnitudes corresponds exactly to a brightness ratio of 100:1. In addition, the magnitudes of stars are decimalized; for example, a star isn’t just a “second-magnitude star,” it has a magnitude of 2.0 (or 2.1, 2.3, and so forth). So what number is it that, when multiplied together five times, gives you this factor of 100? Play on your calculator and see if you can get it. The answer turns out to be about 2.5, which is the fifth root of 100. This means that a magnitude 1.0 star and a magnitude 2.0 star differ in brightness by a factor of about 2.5. Likewise, we receive about 2.5 times as much light from a magnitude 2.0 star as from a magnitude 3.0 star. What about the difference between a magnitude 1.0

star and a magnitude 3.0 star? Since the difference is 2.5 times for each “step” of magnitude, the total difference in brightness is 2.5 × 2.5 = 6.25 times.

Here are a few rules of thumb that might help those new to this system. If two stars differ by 0.75 magnitudes, they differ by a factor of about 2 in brightness. If they are 2.5 magnitudes apart, they differ in brightness by a factor of 10, and a 4-magnitude difference corresponds to a difference in brightness of a factor of 40.You might be saying to yourself at this point, “Why do astronomers continue to use this complicated system from more than 2000 years ago?” That’s an excellent question and, as we shall discuss, astronomers today can use other ways of expressing how bright a star looks. But because this system is still used in many books, star charts, and computer apps, we felt we had to introduce students to it (even though we were very tempted to leave it out.)

The brightest stars, those that were traditionally referred to as first-magnitude stars, actually turned out (when measured accurately) not to be identical in brightness. For example, the brightest star in the sky, Sirius, sends us about 10 times as much light as the average first-magnitude star. On the modern magnitude scale, Sirius, the star with the brightest apparent magnitude, has been assigned a magnitude of −1.5. Other objects in the sky can appear even brighter. Venus at its brightest is of magnitude −4.4, while the Sun has a magnitude of −26.8. [link] shows the range of observed magnitudes from the brightest to the faintest, along with the actual magnitudes of several well-known objects. The important fact to remember when using magnitude is that the system goes backward: the *larger* the magnitude, the *fainter* the object you are observing.

Apparent Magnitudes of Well-Known Objects.



The faintest magnitudes that can be detected by the unaided eye, binoculars, and large telescopes are also shown.

Example:

The Magnitude Equation

Even scientists can't calculate fifth roots in their heads, so astronomers have summarized the above discussion in an equation to help calculate the difference in brightness for stars with different magnitudes. If m_1 and m_2 are the magnitudes of two stars, then we can calculate the ratio of their brightness $\left(\frac{b_2}{b_1}\right)$ using this equation:

Equation:

$$m_1 - m_2 = 2.5 \log\left(\frac{b_2}{b_1}\right) \qquad \text{or} \qquad \frac{b_2}{b_1} = 2.5^{m_1 - m_2}$$

Here is another way to write this equation:

Equation:

$$\frac{b_2}{b_1} = \left(100^{0.2}\right)^{m_1 - m_2}$$

Let’s do a real example, just to show how this works. Imagine that an astronomer has discovered something special about a dim star (magnitude 8.5), and she wants to tell her students how much dimmer the star is than Sirius. Star 1 in the equation will be our dim star and star 2 will be Sirius.

Solution

Remember, Sirius has a magnitude of −1.5. In that case:

Equation:

$$\begin{aligned}\frac{b_2}{b_1} &= \left(100^{0.2}\right)^{8.5-(-1.5)} = \left(100^{0.2}\right)^{10} \\ &= (100)^2 = 100 \times 100 = 10{,}000\end{aligned}$$

Any of the preceding examples is straightforward to do with a scientific calculator, but one always wants to be able to check answers with a rough approximation done in one’s head (or on paper) to make sure one did not drop a decimal point or a minus sign somewhere. What gets in the way of such mental checks for these sorts of problems is the gnarly process of taking ten to the 1/5 th power,

$$10^{0.2}$$

, so we will perform that operation once and for all and use its value 2.5, or 2.512 if we need more precision (or even 2.5118864315 if one wants six decimal place accuracy).

2.512^1	**2.51**
2.512^1	**2.51**
2.512^2	6.31
2.512^3	15.85
2.512^4	39.81
2.512^5	100
2.512^6	251
2.512^7	631
2.512^8	1585
2.512^9	3981

2.512^1	**2.51**
2.512^{10}	10000
(10,1)	(10,2)

Use your pattern recognition skills to compare the first half of the table to the second half, and extend it to

$$2.512^{15}$$

.

Let us now apply it to the common misconception that Polaris (magnitude 2.0) is the brightest star in the sky, a distinction actually belongs to Sirius (magnitude −1.5). The brightness ratio is

$$\frac{b_{Sirius}}{b_{Polaris}} = \left(10^{0.2}\right)^{2.0-(-1.5)} \Rightarrow 2.512^{2.0-(-1.5)} = 2.512^{3.5}$$

If the exponent of 2.512 had been 3, our table would have told us that the ratio of brightnesses would have been about 16 or if the exponent of 2.512 had been 4, our table would have told us that the ratio of brightnesses would have been about 40. Since the exponent of 2.512 is half way between these two, the ratio of brightnesses must be somewhat in between 16 and 40. Because the exponetiation is not linear, we would not have expected the result to be precisely their average, 28, but is likely pretty close. Indeed, a friend you are talking to about this is not likely to care whether your declaration is off by 10%, which it is.

Use of this table is as simple as that. If the exponent of 2.512 had been 3.8, our table and your intuition would but the brightness at something like 35.

Those of you with a basic calculator can simply use this method to get an answer good enough to about 10%.

Can this technique be used to get a more precise answer? Yes is one can form the secondary table

$2.512^{0.1}$	**1.1**
$2.512^{0.2}$	1.2
$2.512^{0.3}$	1.32
$2.512^{0.4}$	1.45
$2.512^{0.5}$	1.58
$2.512^{0.6}$	1.74
$2.512^{0.7}$	1.91

$2.512^{0.1}$	**1.1**
$2.512^{0.8}$	2.09
$2.512^{0.9}$	2.29

and use the relation for exponents that

$$x^{a+b} = x^a x^b$$

to give

$$\frac{b_{Sirius}}{b_{Polaris}} = 2.512^{3.5} = 2.512^3 \times 2.512^{0.5} = 15.85 \times 1.58 = 25$$

,
the correct answer.

Note:
Answer:

$$\frac{b_{\text{Sirius}}}{b_{\text{Polaris}}} = \left(100^{0.2}\right)^{2.0-(-1.5)} = \left(100^{0.2}\right)^{3.5} = 100^{0.7} = 25$$

(Hint: If you only have a basic calculator, you may wonder how to take 100 to the 0.7th power. But this is something you can ask Google to do. Google now accepts mathematical questions and will answer them. So try it for yourself. Ask Google, “What is 100 to the 0.7th power?”)

Our calculation shows that Sirius’ apparent brightness is 25 times greater than Polaris’ apparent brightness.

Other Units of Brightness

Although the magnitude scale is still used for visual astronomy, it is not used at all in newer branches of the field. In radio astronomy, for example, no equivalent of the magnitude system has been defined. Rather, radio astronomers measure the amount of energy being collected each second by each square meter of a radio telescope and express the brightness of each source in terms of, for example, watts per square meter.

Similarly, most researchers in the fields of infrared, X-ray, and gamma-ray astronomy use energy per area per second rather than magnitudes to express the results of their measurements. Nevertheless, astronomers in all fields are careful to distinguish between the *luminosity* of the source (even when that luminosity is all in X-rays) and the amount of energy that happens to reach us on Earth. After all, the luminosity is a really important characteristic that tells us a lot about the object in question, whereas the energy that reaches Earth is an accident of cosmic geography.

To make the comparison among stars easy, in this text, we avoid the use of magnitudes as much as possible and will express the luminosity of other stars in terms of the Sun’s luminosity. For example, the luminosity of Sirius is 25 times that of the Sun. We use the symbol L_{Sun} to denote the Sun’s luminosity; hence, that of Sirius can be written as 25 L_{Sun}.

Key Concepts and Summary

The total energy emitted per second by a star is called its luminosity. How bright a star looks from the perspective of Earth is its apparent brightness. The apparent brightness of a star depends on both its luminosity and its distance from Earth. Thus, the determination of apparent brightness and measurement of the distance to a star provide enough information to calculate its luminosity. The apparent brightnesses of stars are often expressed in terms of magnitudes, which is an old system based on how human vision interprets relative light intensity.

Glossary

apparent brightness

a measure of the amount of light received by Earth from a star or other object—that is, how bright an object appears in the sky, as contrasted with its luminosity

luminosity

the rate at which a star or other object emits electromagnetic energy into space; the total power output of an object

magnitude

an older system of measuring the amount of light we receive from a star or other luminous object; the larger the magnitude, the less radiation we receive from the object

Derived copy of Using Spectra to Measure Stellar Radius, Composition, and Motion

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Understand how astronomers can learn about a star’s radius and composition by studying its spectrum
- Explain how astronomers can measure the motion and rotation of a star using the Doppler effect
- Describe the proper motion of a star and how it relates to a star’s space velocity

Analyzing the spectrum of a star can teach us all kinds of things in addition to its temperature. We can measure its detailed chemical composition as well as the pressure in its atmosphere. From the pressure, we get clues about its size. We can also measure its motion toward or away from us and estimate its rotation.

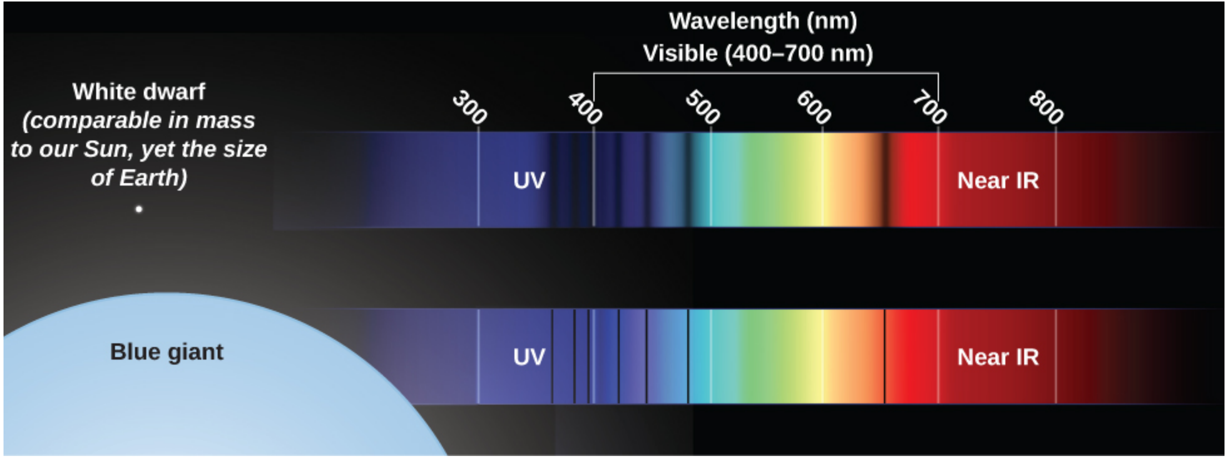
Clues to the Size of a Star

As we shall see in The Stars: A Celestial Census, stars come in a wide variety of sizes. At some periods in their lives, stars can expand to enormous dimensions. Stars of such exaggerated size are called **giants**. Luckily for the astronomer, stellar spectra can be used to distinguish giants from run-of-the-mill stars (such as our Sun).

Suppose you want to determine whether a star is a giant. A giant star has a large, extended photosphere. Because it is so large, a giant star’s atoms are spread over a great volume, which means that the density of particles in the star’s photosphere is low. As a result, the pressure in a giant star’s photosphere is also low. This low pressure affects the spectrum in two ways. First, a star with a lower-pressure photosphere shows narrower spectral lines than a star of the same temperature with a higher-pressure photosphere ([link]). The difference is large enough that careful study of spectra can tell which of two stars at the same temperature has a higher pressure (and is thus more compressed) and which has a lower pressure (and thus must be extended). This effect is due to collisions between particles in the star’s photosphere—more collisions lead to broader spectral lines. Collisions will, of course, be more frequent in a higher-density environment. Think about it like traffic—collisions are much more likely during rush hour, when the density of cars is high.

Second, more atoms are ionized in a giant star than in a star like the Sun with the same temperature. The ionization of atoms in a star’s outer layers is caused mainly by photons, and the amount of energy carried by photons is determined by temperature. But how long atoms *stay* ionized depends in part on pressure. Compared with what happens in the Sun (with its relatively dense photosphere), ionized atoms in a giant star’s photosphere are less likely to pass close enough to electrons to interact and combine with one or more of them, thereby becoming neutral again. Ionized atoms, as we discussed earlier, have different spectra from atoms that are neutral.

Spectral Lines.



This figure illustrates one difference in the spectral lines from stars of the same temperature but different pressures. A giant star with a very-low-pressure photosphere shows very narrow spectral lines (bottom), whereas a smaller star with a higher-pressure photosphere shows much broader spectral lines (top). (credit: modification of work by NASA, ESA, A. Field, and J. Kalirai (STScI))

Abundances of the Elements

Absorption lines of a majority of the known chemical elements have now been identified in the spectra of the Sun and stars. If we see lines of iron in a star’s spectrum, for example, then we know immediately that the star must contain iron.

Note that the *absence* of an element’s spectral lines does not necessarily mean that the element itself is absent. As we saw, the temperature and pressure in a star’s atmosphere will determine what types of atoms are able to produce absorption lines. Only if the physical conditions in a star’s photosphere are such that lines of an element *should* (according to calculations) be there can we conclude that the absence of observable spectral lines implies low abundance of the element.

Suppose two stars have identical temperatures and pressures, but the lines of, say, sodium are stronger in one than in the other. Stronger lines mean that there are more atoms in the stellar photosphere absorbing light. Therefore, we know immediately that the star with stronger sodium lines contains more sodium. Complex calculations are required to determine exactly how much more, but those calculations can be done for any element observed in any star with any temperature and pressure.

Of course, astronomy textbooks such as ours always make these things sound a bit easier than they really are. If you look at the stellar spectra such as those in [link], you may get some feeling for how hard it is to decode all of the information contained in the thousands of absorption lines. First of all, it has taken many years of careful laboratory work on Earth to determine the precise wavelengths at which hot gases of each element have their spectral lines. Long books and computer databases have been compiled to show the lines of each element that can be seen at each temperature. Second, stellar spectra usually have many lines from a number of elements, and we must be careful to sort them out correctly. Sometimes nature is unhelpful, and lines of different elements have identical wavelengths, thereby adding to the confusion. And third, as we saw in the chapter on Radiation and Spectra, the motion of the star can change the observed wavelength of each of the lines. So, the observed wavelengths may not match laboratory measurements exactly. In practice, analyzing stellar spectra is a demanding, sometimes frustrating task that requires both training and skill.

Studies of stellar spectra have shown that hydrogen makes up about three-quarters of the mass of most stars. Helium is the second-most abundant element, making up almost a quarter of a star's mass. Together, hydrogen and helium make up from 96 to 99% of the mass; in some stars, they amount to more than 99.9%. Among the 4% or less of "heavy elements," oxygen, carbon, neon, iron, nitrogen, silicon, magnesium, and sulfur are among the most abundant. Generally, but not invariably, the elements of lower atomic weight are more abundant than those of higher atomic weight.

Take a careful look at the list of elements in the preceding paragraph. Two of the most abundant are hydrogen and oxygen (which make up water); add carbon and nitrogen and you are starting to write the prescription for the chemistry of an astronomy student. We are made of elements that are common in the universe—just mixed together in a far more sophisticated form (and a much cooler environment) than in a star.

As we mentioned in The Spectra of Stars (and Brown Dwarfs) section, astronomers use the term "metals" to refer to all elements heavier than hydrogen and helium. The fraction of a star's mass that is composed of these elements is referred to as the star's *metallicity*. The metallicity of the Sun, for example, is 0.02, since 2% of the Sun's mass is made of elements heavier than helium.

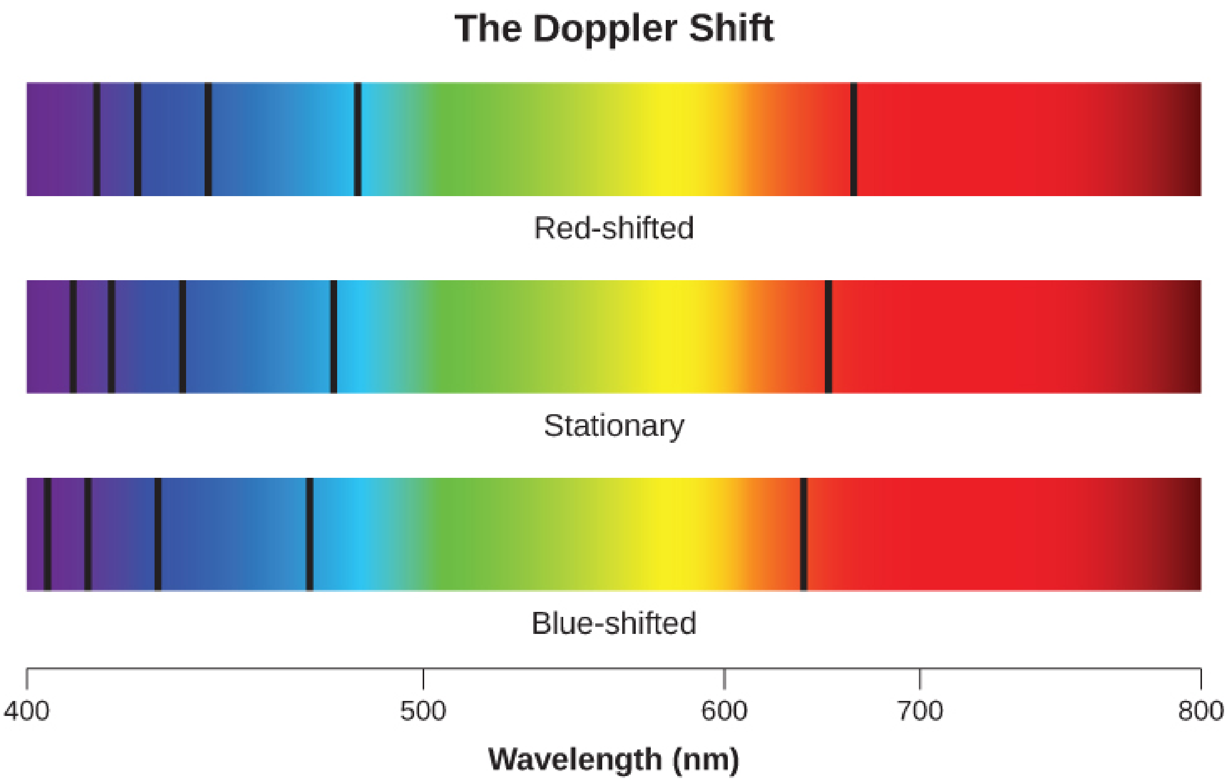
Appendix K lists how common each element is in the universe (compared to hydrogen); these estimates are based primarily on investigation of the

Sun, which is a typical star. Some very rare elements, however, have not been detected in the Sun. Estimates of the amounts of these elements in the universe are based on laboratory measurements of their abundance in primitive meteorites, which are considered representative of unaltered material condensed from the solar nebula (see the Cosmic Samples and the Origin of the Solar System chapter).

Radial Velocity

When we measure the spectrum of a star, we determine the wavelength of each of its lines. If the star is not moving with respect to the Sun, then the wavelength corresponding to each element will be the same as those we measure in a laboratory here on Earth. But if stars are moving toward or away from us, we must consider the *Doppler effect* (see The **Doppler** Effect section). We should see all the spectral lines of moving stars shifted toward the red end of the spectrum if the star is moving away from us, or toward the blue (violet) end if it is moving toward us ([link]). The greater the shift, the faster the star is moving. Such motion, along the line of sight between the star and the observer, is called **radial velocity** and is usually measured in kilometers per second.

Doppler-Shifted Stars.



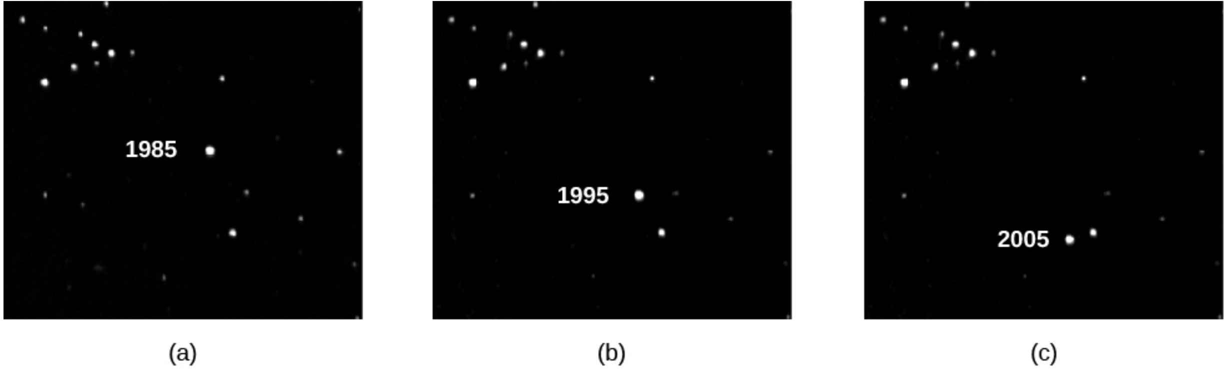
When the spectral lines of a moving star shift toward the red end of the spectrum, we know that the star is moving away from us. If they shift toward the blue end, the star is moving toward us.

William Huggins, pioneering yet again, in 1868 made the first radial velocity determination of a star. He observed the Doppler shift in one of the hydrogen lines in the spectrum of Sirius and found that this star is moving toward the solar system. Today, radial velocity can be measured for any star bright enough for its spectrum to be observed. As we will see in The Stars: A Celestial Census, radial velocity measurements of double stars are crucial in deriving stellar masses.

Proper Motion

There is another type of motion stars can have that cannot be detected with stellar spectra. Unlike radial motion, which is along our line of sight (i.e., toward or away from Earth), this motion, called **proper motion**, is *transverse*: that is, across our line of sight. We see it as a change in the relative positions of the stars on the celestial sphere ([link]). These changes are very slow. Even the star with the largest proper motion takes 200 years to change its position in the sky by an amount equal to the width of the full Moon, and the motions of other stars are smaller yet.

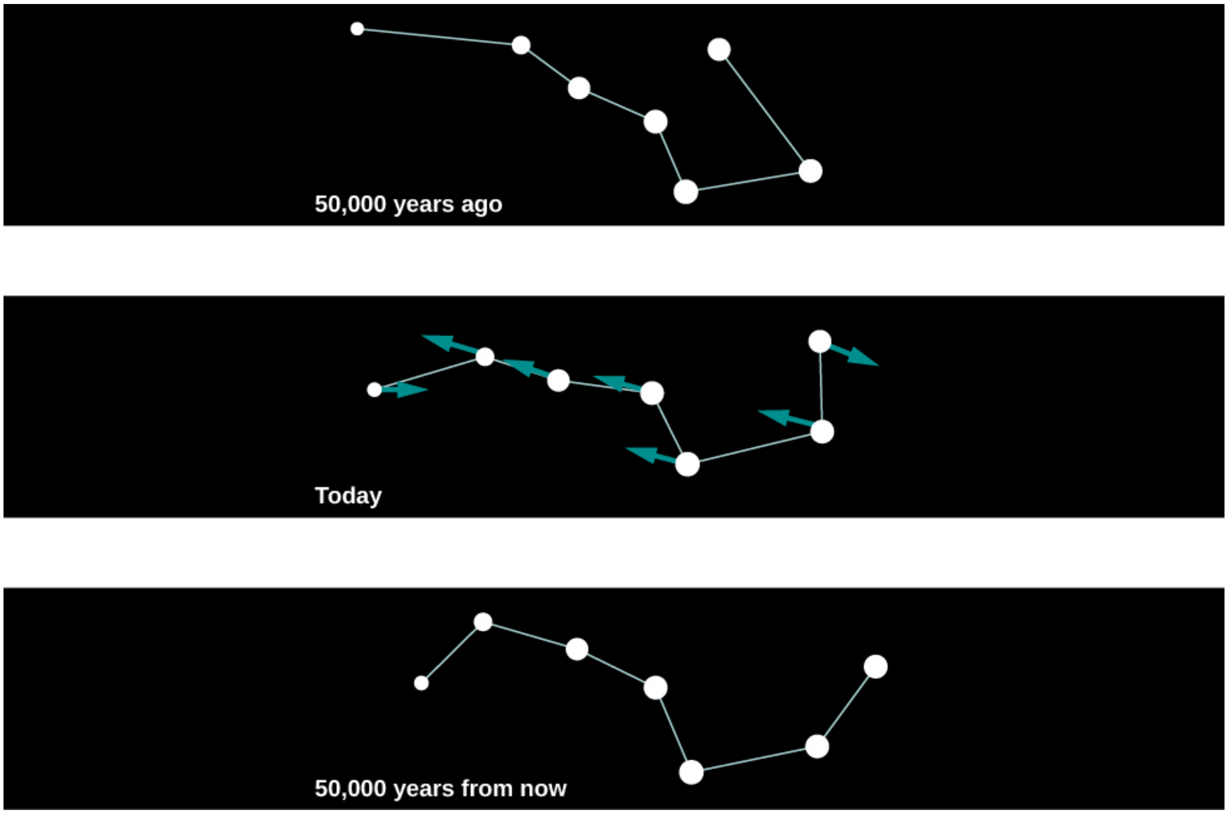
Large Proper Motion.



Three photographs of Barnard’s star, the star with the largest known proper motion, show how this faint star has moved over a period of 20 years. (modification of work by Steve Quirk)

For this reason, with our naked eyes, we do not notice any change in the positions of the bright stars during the course of a human lifetime. If we could live long enough, however, the changes would become obvious. For example, some 50,000 years from now, terrestrial observers will find the handle of the Big Dipper unmistakably more bent than it is now ([link]).

Changes in the Big Dipper.

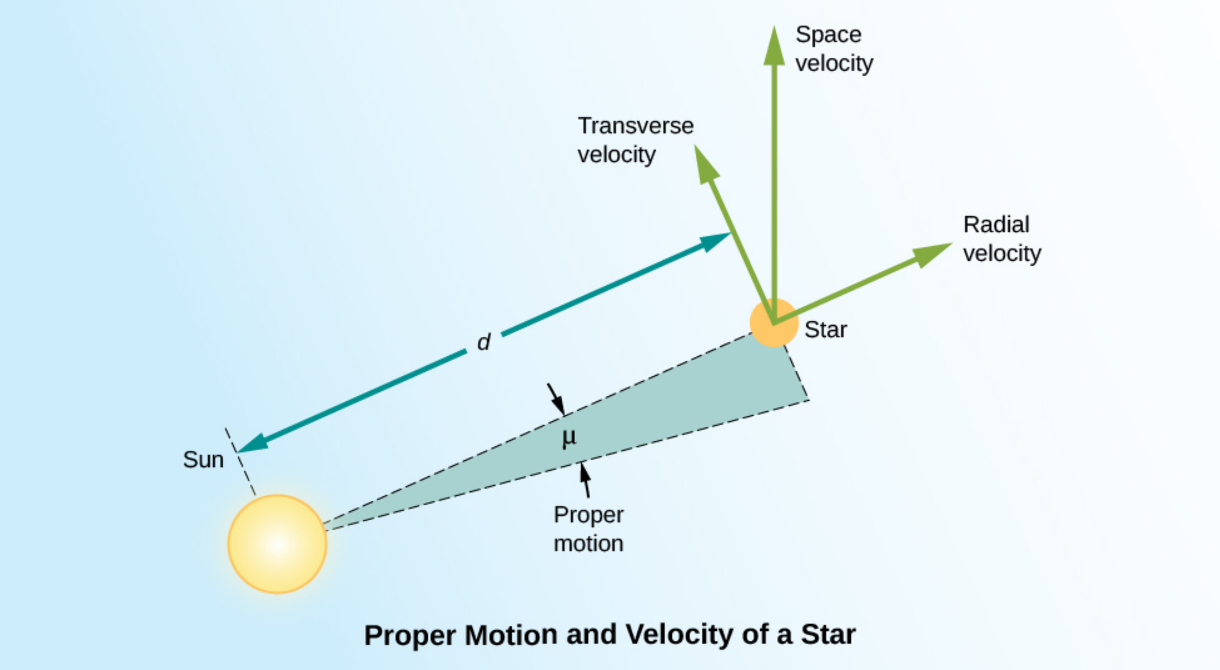


This figure shows changes in the appearance of the Big Dipper due to proper motion of the stars over 100,000 years.

We measure the proper motion of a star in arcseconds (1/3600 of a degree) per year. That is, the measurement of proper motion tells us only by how much of an angle a star has changed its position on the celestial sphere. If two stars at different distances are moving at the same velocity perpendicular to our line of sight, the closer one will show a larger shift in its position on the celestial sphere in a year’s time. As an analogy, imagine you are standing at the side of a freeway. Cars will appear to whiz past you. If you then watch the traffic from a vantage point half a mile away, the cars will move much more slowly across your field of vision. In order to convert this angular motion to a velocity, we need to know how far away the star is.

To know the true **space velocity** of a star—that is, its total speed and the direction in which it is moving through space relative to the Sun—we must know its radial velocity, proper motion, and distance ([link]). A star’s space velocity can also, over time, cause its distance from the Sun to change significantly. Over several hundred thousand years, these changes can be large enough to affect the apparent brightnesses of nearby stars. Today, Sirius, in the constellation Canis Major (the Big Dog) is the brightest star in the sky, but 100,000 years ago, the star Canopus in the constellation Carina (the Keel) was the brightest one. A little over 200,000 years from now, Sirius will have moved away and faded somewhat, and Vega, the bright blue star in Lyra, will take over its place of honor as the brightest star in Earth’s skies.

Space Velocity and Proper Motion.



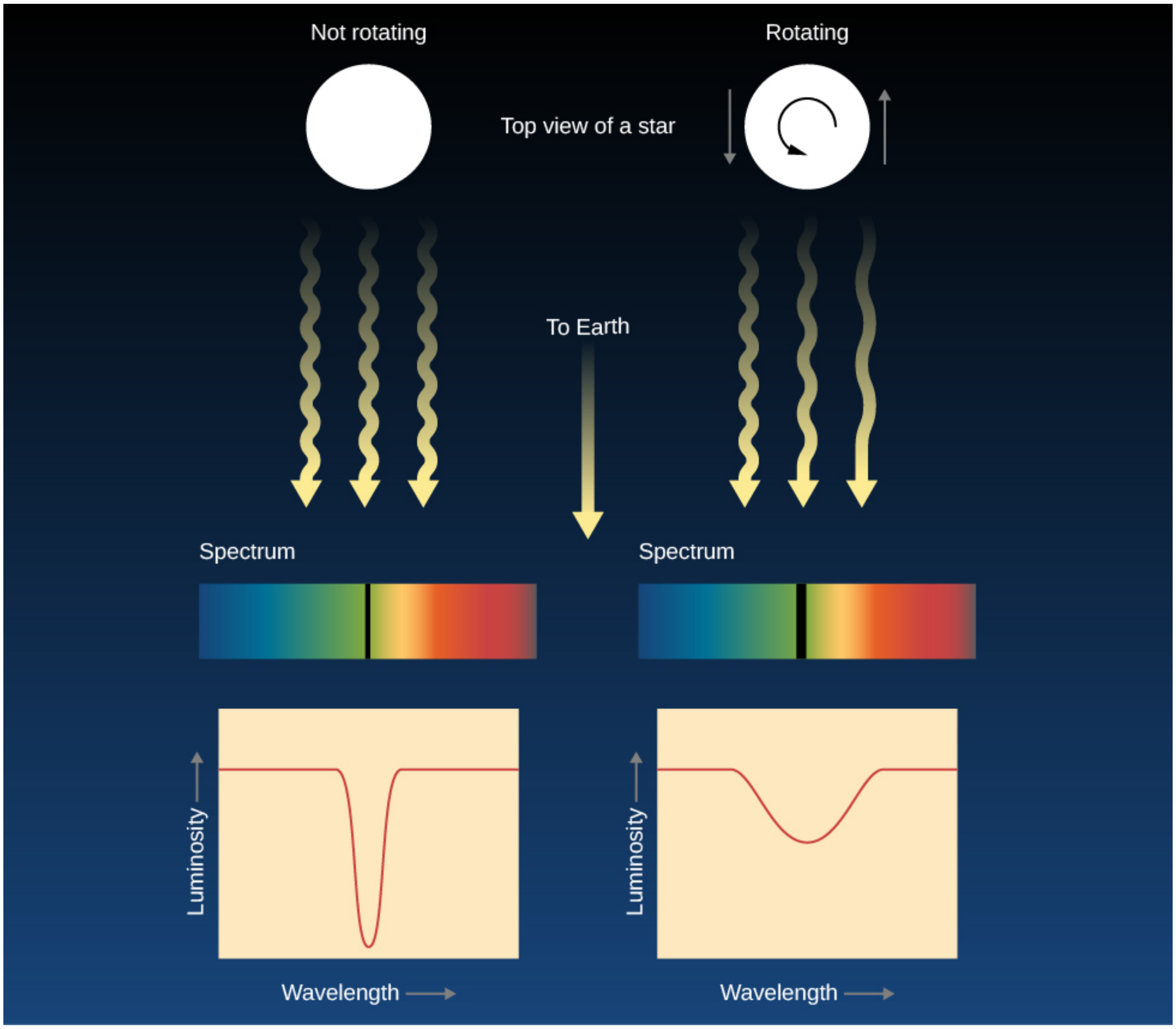
This figure shows the true space velocity of a star. The radial velocity is the component of the space velocity projected along the line of sight from the Sun to a star. The transverse velocity is a component of the space velocity projected on the sky. What astronomers measure is proper motion (μ), which is the change in the apparent direction on the sky measured in fractions of a degree. To convert this change in direction to a speed in, say, kilometers per second, it is necessary to also know the distance (*d*) from the Sun to the star.

Rotation

We can also use the Doppler effect to measure how fast a star rotates. If an object is rotating, then one of its sides is approaching us while the other is receding (unless its axis of rotation happens to be pointed exactly toward us). This is clearly the case for the Sun or a planet; we can observe the light from either the approaching or receding edge of these nearby objects and directly measure the Doppler shifts that arise from the rotation.

Stars, however, are so far away that they all appear as unresolved points. The best we can do is to analyze the light from the entire star at once. Due to the Doppler effect, the lines in the light that come from the side of the star rotating toward us are shifted to shorter wavelengths and the lines in the light from the opposite edge of the star are shifted to longer wavelengths. You can think of each spectral line that we observe as the sum or composite of spectral lines originating from different speeds with respect to us. Each point on the star has its own Doppler shift, so the absorption line we see from the whole star is actually much wider than it would be if the star were not rotating. If a star is rotating rapidly, there will be a greater spread of Doppler shifts and all its spectral lines should be quite broad. In fact, astronomers call this effect *line broadening*, and the amount of broadening can tell us the speed at which the star rotates ([link]).

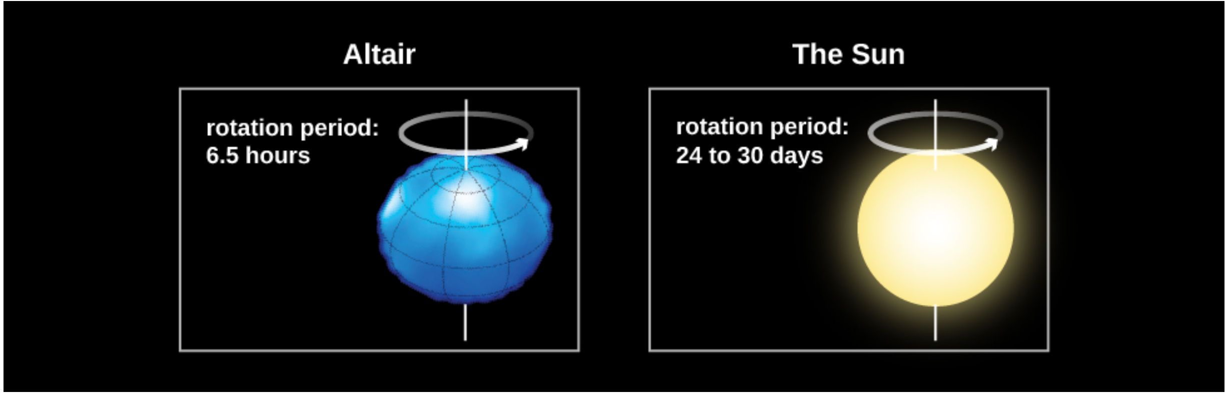
Using a Spectrum to Determine Stellar Rotation.



A rotating star will show broader spectral lines than a nonrotating star.

Measurements of the widths of spectral lines show that many stars rotate faster than the Sun, some with periods of less than a day! These rapid rotators spin so fast that their shapes are “flattened” into what we call *oblate spheroids*. An example of this is the star Vega, which rotates once every 12.5 hours. Vega’s rotation flattens its shape so much that its diameter at the equator is 23% wider than its diameter at the poles ([link]). The Sun, with its rotation period of about a month, rotates rather slowly. Studies have shown that stars decrease their rotational speed as they age. Young stars rotate very quickly, with rotational periods of days or less. Very old stars can have rotation periods of several months.

Comparison of Rotating Stars.



This illustration compares the more rapidly rotating star Altair to the slower rotating Sun.

As you can see, spectroscopy is an extremely powerful technique that helps us learn all kinds of information about stars that we simply could not gather any other way. We will see in later chapters that these same techniques can also teach us about galaxies, which are the most distant objects that can we observe. Without spectroscopy, we would know next to nothing about the universe beyond the solar system.

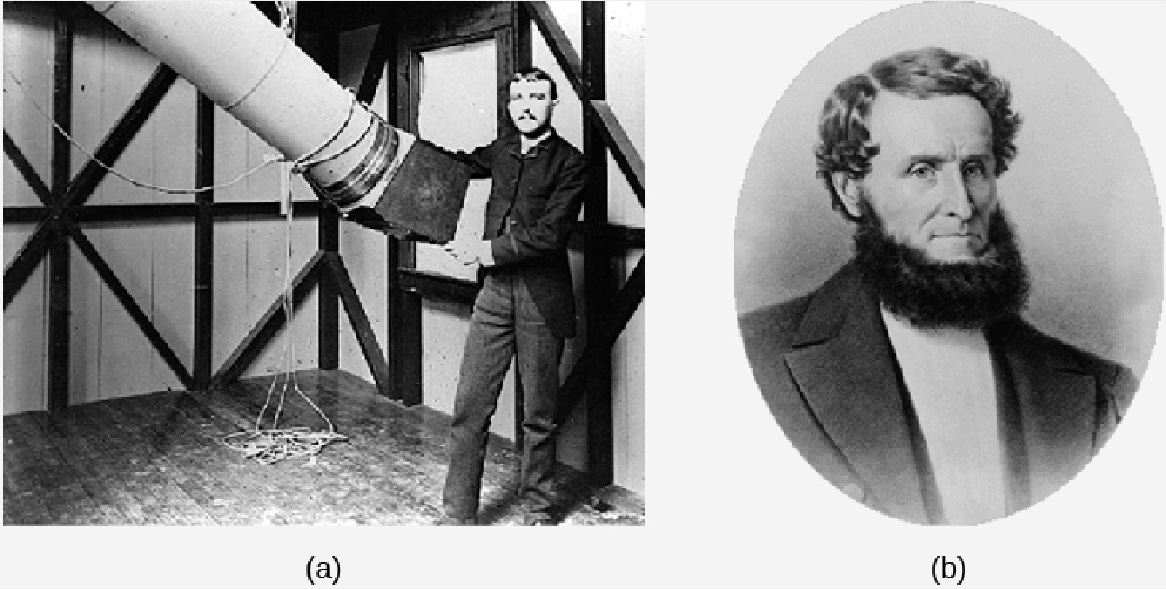
Note:

Astronomy and Philanthropy

Throughout the history of astronomy, contributions from wealthy patrons of the science have made an enormous difference in building new instruments and carrying out long-term research projects. Edward Pickering’s stellar classification project, which was to stretch over several decades, was made possible by major donations from Anna Draper. She was the widow of Henry Draper, a physician who was one of the most accomplished amateur astronomers of the nineteenth century and the first person to successfully photograph the spectrum of a star. Anna Draper gave several hundred thousand dollars to Harvard Observatory. As a result, the great spectroscopic survey is still known as the Henry Draper Memorial, and many stars are still referred to by their “HD” numbers in that catalog (such as HD 209458).

In the 1870s, the eccentric piano builder and real estate magnate James Lick ([link]) decided to leave some of his fortune to build the world’s largest telescope. When, in 1887, the pier to house the telescope was finished, Lick’s body was entombed in it. Atop the foundation rose a 36-inch refractor, which for many years was the main instrument at the Lick Observatory near San Jose.

Henry Draper (1837–1882) and James Lick (1796–1876).



(a) Draper stands next to a telescope used for photography. After his death, his widow funded further astronomy work in his name. (b) Lick was a philanthropist who provided funds to build a 36-inch refractor not only as a memorial to himself but also to aid in further astronomical research.

The Lick telescope remained the largest in the world until 1897, when George Ellery Hale persuaded railroad millionaire Charles Yerkes to finance the construction of a 40-inch telescope near Chicago. More recently, Howard Keck, whose family made its fortune in the oil industry, gave $70 million from his family foundation to the California Institute of Technology to help build the world’s largest telescope atop the 14,000-foot peak of Mauna Kea in Hawaii (see the chapter on Astronomical Instruments to learn more about these telescopes). The Keck Foundation was so pleased with what is now called the Keck telescope that they gave $74 million more to build Keck II, another 10-meter reflector on the same volcanic peak.

Now, if any of you become millionaires or billionaires, and astronomy has sparked your interest, do keep an astronomical instrument or project in mind as you plan your estate. But frankly, private philanthropy could not possibly support the full enterprise of scientific research in astronomy. Much of our exploration of the universe is financed by federal agencies such as the National Science Foundation and NASA in the United States, and by similar government agencies in the other countries. In this way, all of us, through a very small share of our tax dollars, are philanthropists for astronomy.

Key Concepts and Summary

Spectra of stars of the same temperature but different atmospheric pressures have subtle differences, so spectra can be used to determine whether a star has a large radius and low atmospheric pressure (a giant star) or a small radius and high atmospheric pressure. Stellar spectra can also be used to determine the chemical composition of stars; hydrogen and helium make up most of the mass of all stars. Measurements of line shifts produced by the Doppler effect indicate the radial velocity of a star. Broadening of spectral lines by the Doppler effect is a measure of rotational velocity. A star can also show proper motion, due to the component of a star’s space velocity across the line of sight.

For Further Exploration

Articles

Berman, B. “Magnitude Cum Laude.” *Astronomy* (December 1998): 92. How we measure the apparent brightnesses of stars is discussed.

Dvorak, J. “The Women Who Created Modern Astronomy [including Annie Cannon].” *Sky & Telescope* (August 2013): 28.

Hearnshaw, J. “Origins of the Stellar Magnitude Scale.” *Sky & Telescope* (November 1992): 494. A good history of how we have come to have this cumbersome system is discussed.

Hirshfeld, A. “The Absolute Magnitude of Stars.” *Sky & Telescope* (September 1994): 35.

Kaler, J. “Stars in the Cellar: Classes Lost and Found.” *Sky & Telescope* (September 2000): 39. An introduction is provided for spectral types and the new classes L and T.

Kaler, J. “Origins of the Spectral Sequence.” *Sky & Telescope* (February 1986): 129.

Skrutskie, M. “2MASS: Unveiling the Infrared Universe.” *Sky & Telescope* (July 2001): 34. This article focuses on an all-sky survey at 2 microns.

Sneden, C. “Reading the Colors of the Stars.” *Astronomy* (April 1989): 36. This article includes a discussion of what we learn from spectroscopy.

Steffey, P. “The Truth about Star Colors.” *Sky & Telescope* (September 1992): 266. The color index and how the eye and film “see” colors are discussed.

Tomkins, J. “Once and Future Celestial Kings.” *Sky & Telescope* (April 1989): 59. Calculating the motion of stars and determining which stars were, are, and will be brightest in the sky are discussed.

Websites

Discovery of Brown Dwarfs:
http://w.astro.berkeley.edu/~basri/bdwarfs/SciAm-book.pdf.

Listing of Nearby Brown Dwarfs: http://www.solstation.com/stars/pc10bd.htm.

Spectral Types of Stars: http://www.skyandtelescope.com/astronomy-equipment/the-spectral-types-of-stars/.

Stellar Velocities https://www.e-education.psu.edu/astro801/content/l4_p7.html.

Unheard Voices! The Contributions of Women to Astronomy: A Resource Guide: http://multiverse.ssl.berkeley.edu/women and http://www.astrosociety.org/education/astronomy-resource-guides/women-in-astronomy-an-introductory-resource-guide/.

Videos

When You Are Just Too Small to be a Star: https://www.youtube.com/watch?v=zXCDsb4n4KU. 2013 Public Talk on Brown Dwarfs and Planets by Dr. Gibor Basri of the University of California–Berkeley (1:32:52).

Collaborative Group Activities

A. The Voyagers in Astronomy feature on Annie Cannon: Classifier of the Stars discusses some of the difficulties women who wanted to do astronomy faced in the first half of the twentieth century. What does your group think about the situation for women today? Do men and women have an equal chance to become scientists? Discuss with your group whether, in your experience, boys and girls were equally encouraged to do science and math where you went to school.
B. In the section on magnitudes in The Brightness of Stars, we discussed how this old system of classifying how bright different stars appear to the eye first developed. Your authors complained about the fact that this old system still has to be taught to every generation of new students. Can your group think of any other traditional systems of doing things in science and measurement where tradition rules even

though common sense says a better system could certainly be found. Explain. (Hint: Try Daylight Savings Time, or metric versus English units.)

C. Suppose you could observe a star that has only one spectral line. Could you tell what element that spectral line comes from? Make a list of reasons with your group about why you answered yes or no.
D. A wealthy alumnus of your college decides to give $50 million to the astronomy department to build a world-class observatory for learning more about the characteristics of stars. Have your group discuss what kind of equipment they would put in the observatory. Where should this observatory be located? Justify your answers. (You may want to refer back to the Astronomical Instruments chapter and to revisit this question as you learn more about the stars and equipment for observing them in future chapters.)
E. For some astronomers, introducing a new spectral type for the stars (like the types L, T, and Y discussed in the text) is similar to introducing a new area code for telephone calls. No one likes to disrupt the old system, but sometimes it is simply necessary. Have your group make a list of steps an astronomer would have to go through to persuade colleagues that a new spectral class is needed.

Review Questions

Exercise:

Problem:

What two factors determine how bright a star appears to be in the sky?

Exercise:

Problem: Explain why color is a measure of a star’s temperature.

Exercise:

Problem:

What is the main reason that the spectra of all stars are not identical? Explain.

Exercise:

Problem:

What elements are stars mostly made of? How do we know this?

Exercise:

Problem:

What did Annie Cannon contribute to the understanding of stellar spectra?

Exercise:

Problem:

Name five characteristics of a star that can be determined by measuring its spectrum. Explain how you would use a spectrum to determine these characteristics.

Exercise:

Problem:

How do objects of spectral types L, T, and Y differ from those of the other spectral types?

Exercise:

Problem:

Do stars that look brighter in the sky have larger or smaller magnitudes than fainter stars?

Exercise:

Problem:

The star Antares has an apparent magnitude of 1.0, whereas the star Procyon has an apparent magnitude of 0.4. Which star appears brighter in the sky?

Exercise:

Problem:

Based on their colors, which of the following stars is hottest? Which is coolest? Archenar (blue), Betelgeuse (red), Capella (yellow).

Exercise:

Problem: Order the seven basic spectral types from hottest to coldest.

Exercise:

Problem:

What is the defining difference between a brown dwarf and a true star?

Thought Questions

Exercise:

Problem:

If the star Sirius emits 23 times more energy than the Sun, why does the Sun appear brighter in the sky?

Exercise:

Problem:

How would two stars of equal luminosity—one blue and the other red—appear in an image taken through a filter that passes mainly blue light? How would their appearance change in an image taken through a filter that transmits mainly red light?

Exercise:

Problem:

[link] lists the temperature ranges that correspond to the different spectral types. What part of the star do these temperatures refer to? Why?

Exercise:

Problem:

Suppose you are given the task of measuring the colors of the brightest stars, listed in Appendix J, through three filters: the first transmits blue light, the second transmits yellow light, and the third transmits red light. If you observe the star Vega, it will appear equally bright through each of the three filters. Which stars will appear brighter through the blue filter than through the red filter? Which stars will appear brighter through the red filter? Which star is likely to have colors most nearly like those of Vega?

Exercise:

Problem:

Star X has lines of ionized helium in its spectrum, and star Y has bands of titanium oxide. Which is hotter? Why? The spectrum of star Z shows lines of ionized helium and also molecular bands of titanium oxide. What is strange about this spectrum? Can you suggest an explanation?

Exercise:

Problem:

The spectrum of the Sun has hundreds of strong lines of nonionized iron but only a few, very weak lines of helium. A star of spectral type B has very strong lines of helium but very weak iron lines. Do these differences mean that the Sun contains more iron and less helium than the B star? Explain.

Exercise:

Problem:

What are the approximate spectral classes of stars with the following characteristics?

A. Balmer lines of hydrogen are very strong; some lines of ionized metals are present.

B. The strongest lines are those of ionized helium.
C. Lines of ionized calcium are the strongest in the spectrum; hydrogen lines show only moderate strength; lines of neutral and metals are present.
D. The strongest lines are those of neutral metals and bands of titanium oxide.

Exercise:

Problem:

Look at the chemical elements in Appendix K. Can you identify any relationship between the abundance of an element and its atomic weight? Are there any obvious exceptions to this relationship?

Exercise:

Problem:

Appendix I lists some of the nearest stars. Are most of these stars hotter or cooler than the Sun? Do any of them emit more energy than the Sun? If so, which ones?

Exercise:

Problem:

Appendix J lists the stars that appear brightest in our sky. Are most of these hotter or cooler than the Sun? Can you suggest a reason for the difference between this answer and the answer to the previous question? (Hint: Look at the luminosities.) Is there any tendency for a correlation between temperature and luminosity? Are there exceptions to the correlation?

Exercise:

Problem:

What star appears the brightest in the sky (other than the Sun)? The second brightest? What color is Betelgeuse? Use Appendix J to find the answers.

Exercise:

Problem:

Suppose hominids one million years ago had left behind maps of the night sky. Would these maps represent accurately the sky that we see today? Why or why not?

Exercise:

Problem:

Why can only a lower limit to the rate of stellar rotation be determined from line broadening rather than the actual rotation rate? (Refer to [link].)

Exercise:

Problem:

Why do you think astronomers have suggested three different spectral types (L, T, and Y) for the brown dwarfs instead of M? Why was one not enough?

Exercise:

Problem:

Sam, a college student, just bought a new car. Sam’s friend Adam, a graduate student in astronomy, asks Sam for a ride. In the car, Adam remarks that the colors on the temperature control are wrong. Why did he say that?

(credit: modification of work by Michael Sheehan)

Exercise:

Problem:

Would a red star have a smaller or larger magnitude in a red filter than in a blue filter?

Exercise:

Problem:

Two stars have proper motions of one arcsecond per year. Star A is 20 light-years from Earth, and Star B is 10 light-years away from Earth. Which one has the faster velocity in space?

Exercise:

Problem:

Suppose there are three stars in space, each moving at 100 km/s. Star A is moving across (i.e., perpendicular to) our line of sight, Star B is moving directly away from Earth, and Star C is moving away from Earth, but at a 30° angle to the line of sight. From which star will you observe the greatest Doppler shift? From which star will you observe the smallest Doppler shift?

Exercise:

Problem:

What would you say to a friend who made this statement, “The visible-light spectrum of the Sun shows weak hydrogen lines and strong calcium lines. The Sun must therefore contain more calcium than hydrogen.”?

Figuring for Yourself

Exercise:

Problem:

In Appendix J, how much more luminous is the most luminous of the stars than the least luminous?

For [link] through [link], use the equations relating magnitude and apparent brightness given in the section on the magnitude scale in The Brightness of Stars and [link].

Exercise:

Problem:

Verify that if two stars have a difference of five magnitudes, this corresponds to a factor of 100 in the ratio $\left(\frac{b_2}{b_1}\right)$; that 2.5 magnitudes corresponds to a factor of 10; and that 0.75 magnitudes corresponds to a factor of 2.

Exercise:

Problem:

As seen from Earth, the Sun has an apparent magnitude of about −26.7. What is the apparent magnitude of the Sun as seen from Saturn, about 10 AU away? (Remember that one AU is the distance from Earth to the Sun and that the brightness decreases as the inverse square of the distance.) Would the Sun still be the brightest star in the sky?

Exercise:

Problem:

An astronomer is investigating a faint star that has recently been discovered in very sensitive surveys of the sky. The star has a magnitude of 16. How much less bright is it than Antares, a star with magnitude roughly equal to 1?

Exercise:

Problem:

The center of a faint but active galaxy has magnitude 26. How much less bright does it look than the very faintest star that our eyes can see, roughly magnitude 6?

Exercise:

Problem:

You have enough information from this chapter to estimate the distance to Alpha Centauri, the second nearest star, which has an apparent magnitude of 0. Since it is a G2 star, like the Sun, assume it has the same luminosity as the Sun and the difference in magnitudes is a result only of the difference in distance. Estimate how far away Alpha Centauri is. Describe the necessary steps in words and then do the calculation. (As we will learn in the Celestial Distances chapter, this method—namely, assuming that stars with identical spectral types emit the same amount of energy—is actually used to estimate distances to stars.) If you assume the distance to the Sun is in AU, your answer will come out in AU.

Exercise:

Problem:

Do the previous problem again, this time using the information that the Sun is 150,000,000 km away. You will get a very large number of km as your answer. To get a better feeling for how the distances compare, try calculating the time it takes light at a speed of 299,338 km/s to travel from the Sun to Earth and from Alpha Centauri to Earth. For Alpha Centauri, figure out how long the trip will take in years as well as in seconds.

Exercise:

Problem:

Star A and Star B have different apparent brightnesses but identical luminosities. If Star A is 20 light-years away from Earth and Star B is 40 light-years away from Earth, which star appears brighter and by what factor?

Exercise:

Problem:

Star A and Star B have different apparent brightnesses but identical luminosities. Star A is 10 light-years away from Earth and appears 36 times brighter than Star B. How far away is Star B?

Exercise:

Problem:

The star Sirius A has an apparent magnitude of −1.5. Sirius A has a dim companion, Sirius B, which is 10,000 times less bright than Sirius A. What is the apparent magnitude of Sirius B? Can Sirius B be seen with the naked eye?

Exercise:

Problem:

Our Sun, a type G star, has a surface temperature of 5800 K. We know, therefore, that it is cooler than a type O star and hotter than a type M star. Given what you learned about the temperature ranges of these types of stars, how many times hotter than our Sun is the hottest type O star? How many times cooler than our Sun is the coolest type M star?

Exercise:

Problem:

The star Sirius A is about 9 light years away from the Sun. If a space ship travels there at v = 0.6 c, how many Earth years does the one-way trip take? How many years pass for the people on the space ship on the one-way trip?

Glossary

giant
    a star of exaggerated size with a large, extended photosphere

proper motion

the angular change per year in the direction of a star as seen from the Sun

radial velocity

motion toward or away from the observer; the component of relative velocity that lies in the line of sight

space velocity

the total (three-dimensional) speed and direction with which an object is moving through space relative to the Sun addition to p 678 new figs

The Cosmological Distance Ladder

Let us actually find the distance to a star using the parallax technique. The bright Southern star Beta Doradus was observed by the Fine Guidance Sensor on the Hubble Space Telescope[footnote] to have a parallax of 0.00314 ±0.000016 arc seconds, so its distance is

G. Fritz Benedict, *et al.*, The Astronomical Journal, Volume 133, Issue 4, pp. 1810-1827 (2007)

D (pc) = 1/parallax (s) = 1/(0.00314 ±0.000059 s) = 318±15 pc.

If we want to understand the *why* of stars as well as the *what*, we must add to its catalogue of extrinsic properties like distance, position in the sky, spectral type, and *apparent brightness* (b), that as seen at Earth. The first step in investigating a star's intrinsic properties is to define its *absolute brightness* or *luminosity* (L), defined as how bright it would be if it were precisely 10 pc away.

As seen in Figure 5.5, brightness falls off with the square of the distance, so the ratio of the luminosity at 10 pc, $L_{@10pc}$, to the *apparent* brightness, b, we see at its true distance will give the ratios of the squares of the star's true distance divided by 10 pc:[footnote]

The subscript is redundant but helpful to keep the first few times one encounters this idea.

$$\frac{L_{@10pc}}{b} = \frac{D^2}{(10pc)^2}$$

.

In Example 17 we presented a relationship between the ratio of brightnesses of two different stars and their *apparent magnitudes* . (The difference of the latter is proportional to the logarithm of the former.) We can use this same relation for the ratio for *absolute luminosity* (L) to the *apparent brightness* (b) of a single star in for the parallel quantities called the *absolute magnitude* (M)and *apparent magnitude* (m). That is

$$m - M = 2.5\log\left(\frac{L}{b}\right)$$

If we know the distance from its parallax, we can use distance ratio above in this expression to give the intrinsic, *absolute magnitude* (M) of any star:

$$M = m - 2.5\log\left(\frac{L}{b}\right) = m - 2.5\log\left(\left(\frac{D}{10pc}\right)^2\right) = m - 5\log\left(\frac{D}{10pc}\right)$$

,

where we have used the property of logarithms that

$$\log\left(a^p\right) = p\log\left(a\right)$$

to simplify the expression.

Beta Doradus is a Cepheid variable star whose apparent visual magnitude (m_V) (seen through a green filter) is shown fluctuating around 3.63 over a full cycle in Fig. A. Thus, its absolute magnitude (M_V) must be

$$M_V = m_V - 5\log\left(\frac{D}{10pc}\right) = 3.63 - 5\log\left(31.8 \pm 1.5\right) = 3.63 - 5\left(1.50 \pm 0.02\right) = -3.88 \pm 0.1$$

.

That is, Beta Doradus would be almost as bright as Venus if it actually were 10 pc away rather than 318.

Annie Jump Cannon was the central figure in establishing a catalogue of extrinsic spectroscopic properties of stars, a scaffold from which those who followed her might be able to garner ideas about the intrinsic properties of stars. Major progress on this road was made by her young colleague Cecilia Helena Payne (later Payne-Gaposchkin, May 10, 1900 – December 7, 1979), in 1925 the first person to earn a Ph.D. in Astronomy from Radcliff (now part of Harvard).

Her doctoral dissertation, published as “Stellar Atmospheres, A Contribution to the Observational Study of High Temperature in the Reversing Layers of Stars” is one of the most highly read doctoral dissertations of all time. Building on the ionization theory developed by Indian physicist Megh Nad Saha, she was able to show that Cannon’s classification system was a measure of stars’ temperatures. She also upended the conventional wisdom that stars’ compositions were roughly in proportion to the abundances on Earth by showing that stars contained roughly a million times more hydrogen than elements heavier than helium, a group of elements collectively referred to as “metals” by astronomers, though of course containing other sorts of heavy elements.

Modern determinations of the absolute magnitude correct for variations in a star’s proportion of such elements, called its “metallicity.” They also correct for other intrinsic effects like a Cepheid’s evolutionary stage as it passes through what is called the instability strip color of the HR diagram, and extrinsic effects of like interstellar dust that reddens and dims the light from a star. All of these effects alter m_V and hence the value we calculate for M_V. With such considerations, David G. Turner[footnote] places the absolute visual magnitude of Beta Doradus at
David G. Turner, Astrophysics and Space Science, Volume 326, Issue 2, pp 219–231, 2010.

$M_V = -3.91 \pm 0.11$.

Once one has calibrated many stars in Cannon’s classification scheme through this means, Cepheid or not, one may reverse the process and find a star’s distance from its absolute magnitude by using the other relation in Example 17,

$$\frac{L_{@10pc}}{b} = \left(100^{0.2}\right)^{m-M} = 2.512^{m-M}$$

.

This slips nicely into our distance relation to give

$$D = 10pc\sqrt{\frac{L_{@10pc}}{b}} = 10pc\sqrt{\left(100^{0.2}\right)^{m-M}} = 10pc\sqrt{2.512^{m-M}} = 10pc * 1.585^{m-M}$$

This is a helpful new rung on a distance ladder for finding distances to stars too far away for parallax measurements and is called *spectroscopic parallax*, that may be applied much further out until it become the stars become too faint to identify their spectral type.

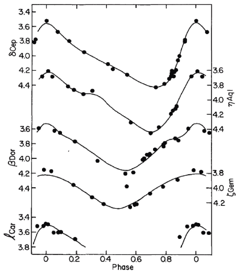


Fig. A. Magnitudes from the fine error sensor, w(FES). Solid curves are light curves through a Visual (green) filter. [footnote]

Edward G. Schmidt and Sidney B. Parsons, The Astrophysical Journal Supplement Series, 48:185-198, 1982. Henrietta S. Leavitt, Annals of Harvard College Observatory 60, 87 (1908); Henrietta Leavitt and Edward C. Pickering, Harvard College Observatory Circular 173 1-3 (1912).

Cepheid Variable stars continue play a central role in cosmology, including questions such as the age of the universe. This is due to the work of another of Annie Jump Cannon's colleagues, Henrietta Leavitt (see Figure 19.10).[footnote] It is not often that nonscientists can catch a glimpse of the creative process of scientific advancement, much less the explication of a natural law as fundamental as then one named for the more famous astronomer, Edwin Hubble, who build upon her work. The following extract is a description of what is coming to be called The Leavitt Law[footnote] in her own words at the time of its discovery:

Henrietta S. Leavitt, Annals of Harvard College Observatory 60, 87 (1908); Henrietta Leavitt and Edward C. Pickering, Harvard College Observatory Circular 173 1-3 (1912)

See for instance, "The Cepheid Period-Luminosity Relation (The Leavitt Law) at Mid-Infrared Wavelengths. II. Second-Epoch LMC Data," Barry F. Madore, Wendy L. Freedman, Jane Rigby, S. E. Persson, Laura Sturch, and Violet Mager, The Astrophysical Journal, Volume 695, Number 2 (2009); "Vetting Galactic Leavitt Law Calibrators Using Radial Velocities: On the Variability, Binarity, and Possible Parallax Error of 19 Long-Period

Cepheids,” Casertano, S. Riess, A. G. Melis, C. Holl, B., The Astrophysical journal Supplement series, Vol.226(2) (2016).

In H.A. 60, No. 4, attention was called to the fact that the brighter variables [2] have the longer periods, but at that time it was felt that the number was too small to warrant the drawing of general conclusions. The periods of 8 additional variables which have been determined since that time, however, conform to the same law.

TABLE I.

PERIODS OF VARIABLE STARS IN THE SMALL MAGELLANIC CLOUD.

H.	**Max.**	**Min.**	**Epoch.**	**Period**	**Res. M.**	**Res. m.**	**H.**	**Max.**	**Min.**	**Ep**
			d,	*d,*						*d,*
1505	14.8	16.1	0.02	1.25336	-0.6	-0.5	1400	14.1	14.8	4.0
1436	14.8	16.4	0.02	1.6637	-0.3	+0.1	*1355*	14.0	14.8	4.8
1446	14.8	16.4	1.38	1.7620	-0.3	+0.1	1374	13.9	15.2	6.0
1506	15.1	16.3	1.08	1.87502	+0.1	+0.1	818	13.6	14.7	4.0
1413	14.7	15.6	0.35	2.17352	-0.2	-0.5	*1610*	13.4	14.6	11.
1460	14.4	15.7	0.00	2.913	-0.3	-0.1	*1365*	13.8	14.8	9.6
1422	14.7	15.9	0.6	3.501	+0.2	+0.2	*1351*	13.4	14.4	4.0
842	14.6	16.1	2.61	4.2897	+0.3	+0.6	827	13.4	14.3	11.
1425	14.3	15.3	2.8	4.547	0.0	-0.1	*822*	13.0	14.6	13.
1742	14.3	15.5	0.95	4.9866	+0.1	+0.2	823	12.2	14.1	2.9
1646	14.4	15.4	4.30	5.311	+0.3	+0.1	824	11.4	12.8	4.
1649	14.3	15.2	5.05	5.323	+0.2	-0.1	821	11.2	12.1	97.
1492	13.8	14.8	0.6	6.2926	-0.2	-0.4				

The relation is shown graphically in Figure 1, in which the abscissas are equal to the periods, expressed in days, and the ordinates are equal to the corresponding magnitudes at maxima and at minima. The two resulting curves, one for maxima and one for minima, are surprisingly smooth, and of remarkable form. In Figure 2, the abscissas are equal to the logarithms of the periods, and the ordinates to the corresponding magnitudes, as in Figure 1.

A straight line can readily be drawn among each of the two series of points corresponding to maxima and minima, thus showing that there is a simple relation between the brightness of the variables and their periods. The logarithm of the period increases by about 0.48 for each increase of one magnitude in brightness. The residuals of the maximum and minimum of each star from the lines in Figure 2 are given in the sixth and seventh columns of Table I. It is possible that the deviations from a straight line may become smaller when an absolute scale of magnitudes is used, and they may even indicate the corrections that need to be applied to the provisional scale. It should be noticed that the average range, for bright and faint [3] variables alike, is about 1.2 magnitudes. Since the variables are probably at nearly the same distance from the Earth, their periods are apparently associated with their actual emission of light, as determined by their mass, density, and surface brightness.

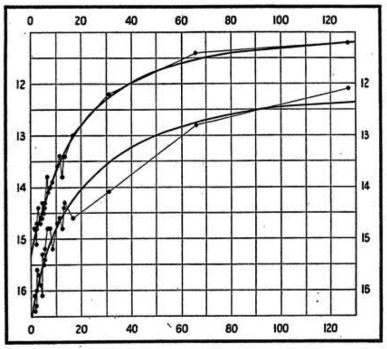
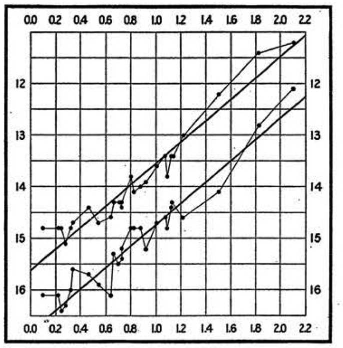


FIG. 1.



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where s is the slope of the line and i is the point (the *y-intercept*) where the line crosses the *y*-axis. We may read this off the upper line of her Fig. 2 as 15.6. Since we are dealing with a graph whose y-axis, the magnitudes, has values that increase downward we must take s as negative. Its size is the reciprocal of "the period [that] increases by about 0.48 for each increase of one magnitude in brightness." Thus setting $y = mmax$ we make explicit one version of Leavitt's Law that is laid out in English in her paper and applied to maximum brightness:

$m\ max = -2.1\ x + 15.6$.

Let us see that we have the correct form for this by applying it at a particular point, say $x = 0.8$:

$m\ max = -2.1\ x + 15.6 = -2.1 \geq 0.8 + 15.6 = 13.92$,

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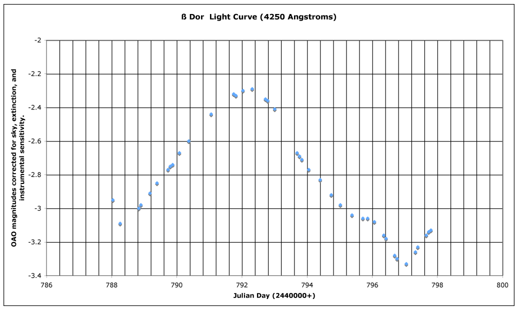

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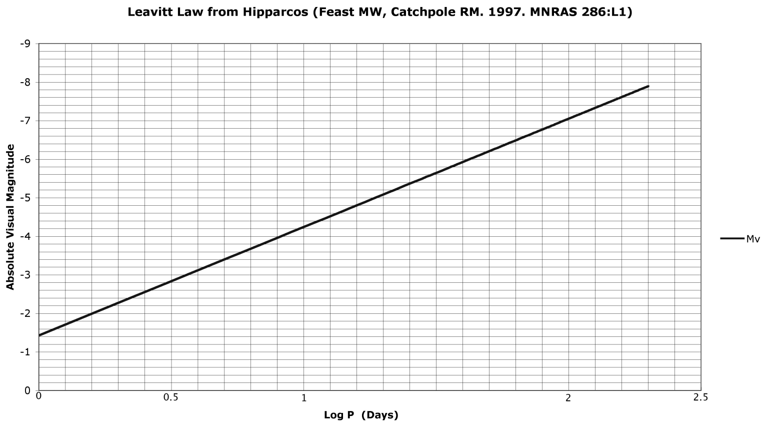
Feast MW, Catchpole RM. 1997. MNRAS 286:L1

$$M\ V = -2.81 \log P - 1.43$$

One may also create a graph of this relation, Fig. C, like Leavitt’s Fig. 2.



The Leavitt Law from Hipparcos satellite parallax from Feast and Catchpole·

To find the absolute magnitudes *MV* of a Cepheid with a 9.8 day period (*Log(9.8) = 0.99)*, one moves upward from *0.99* on the horizontal axis of Fig. C to the line and then moves leftward to the vertical axis, crossing at about *4.2*. This *MV* is a bit larger than Turner's M_V = -3.91 ± 0.11 because is represents an average of many stars. Then the distance to the SMC is

$$D = 10pc * 1.585^{m-M} = 10pc * 1.585^{15.07-(-4.2)} = 10pc * 1.585^{1927} = 10pc * 7154 = 71540pc \pm 500pc$$

Instead of the exponent *m* - *M* = *19.27*, called the *distance modulus*, Scowcroft *et al.* usinga average of 92 such stars give a value of 19.20 ± *0.03*, so we are doing pretty well. They finally correct for reddening due to intervening dust, and average several colors of light, a combined correction of 0.24 magnitudes less, to get their final distance modulus of *18.96± 0.04*., for a distance of *62 kpc* ± *0.3*.

Derived copy of The H–R Diagram and Cosmic Distances

LEARNING OBJECTIVES

By the end of this section, you will be able to:

- Understand how spectral types are used to estimate stellar luminosities
- Examine how these techniques are used by astronomers today

Variable stars are not the only way that we can estimate the luminosity of stars. Another way involves the H–R diagram, which shows that the intrinsic brightness of a star can be estimated if we know its spectral type.

Distances from Spectral Types

As satisfying and productive as variable stars have been for distance measurement, these stars are rare and are not found near all the objects to which we wish to measure distances. Suppose, for example, we need the distance to a star that is not varying, or to a group of stars, none of which is a variable. In this case, it turns out the H–R diagram can come to our rescue.

If we can observe the spectrum of a star, we can estimate its distance from our understanding of the H–R diagram. As discussed in Analyzing Starlight, a detailed examination of a stellar spectrum allows astronomers to classify the star into one of the *spectral types* indicating surface temperature. (The types are O, B, A, F, G, K, M, L, T, and Y; each of these can be divided into numbered subgroups.) In general, however, the spectral type alone is not enough to allow us to estimate luminosity. Look again at [link]. A G2 star could be a main-sequence star with a luminosity of 1 L_{Sun}, or it could be a giant with a luminosity of 100 L_{Sun}, or even a supergiant with a still higher luminosity.

We can learn more from a star’s spectrum, however, than just its temperature. Remember, for example, that we can detect pressure differences in stars from the details of the spectrum. This knowledge is very useful because giant stars are larger (and have lower pressures) than main-sequence stars, and supergiants are still larger than giants. If we look in detail at the spectrum of a star, we can determine whether it is a main-sequence star, a giant, or a supergiant.

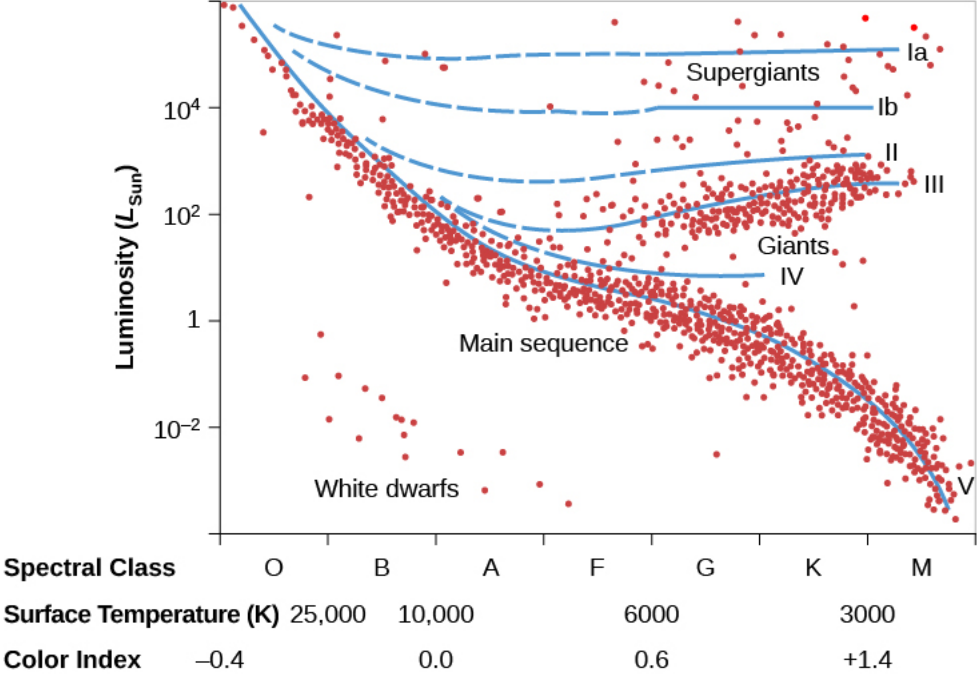
Suppose, to start with the simplest example, that the spectrum, color, and other properties of a distant G2 star match those of the Sun exactly. It is then reasonable to conclude that this distant star is likely to be a main-sequence star just like the Sun and to have the same luminosity as the Sun. But if there are subtle differences between the solar spectrum and the spectrum of the distant star, then the distant star may be a giant or even a supergiant.

The most widely used system of star classification divides stars of a given spectral class into six categories called **luminosity classes**. These luminosity classes are denoted by Roman numbers as follows:

- Ia: Brightest supergiants
- Ib: Less luminous supergiants
- II: Bright giants
- III: Giants
- IV: Subgiants (intermediate between giants and main-sequence stars)
- V: Main-sequence stars

The full spectral specification of a star includes its luminosity class. For example, a main-sequence star with spectral class F3 is written as F3 V. The specification for an M2 giant is M2 III. [link] illustrates the approximate position of stars of various luminosity classes on the H–R diagram. The dashed portions of the lines represent regions with very few or no stars.

Luminosity Classes.



Stars of the same temperature (or spectral class) can fall into different luminosity classes on the Hertzsprung-Russell diagram. By studying details of the spectrum for each star, astronomers can determine which luminosity class they fall in (whether they are main-sequence stars, giant stars, or supergiant stars).

With both its spectral and luminosity classes known, a star’s position on the H–R diagram is uniquely determined. Since the diagram plots luminosity versus temperature, this means we can now read off the star’s luminosity (once its spectrum has helped us place it on the diagram). As before, if we know how luminous the star really is and see how dim it looks, the difference allows us to calculate its distance. (For historical reasons, astronomers sometimes call this method of distance determination *spectroscopic parallax*, even though the method has nothing to do with parallax.)

The H–R diagram method allows astronomers to estimate distances to nearby stars, as well as some of the most distant stars in our Galaxy, but it is anchored by measurements of parallax. The distances measured using parallax are the gold standard for distances: they rely on no assumptions, only geometry. Once astronomers take a spectrum of a nearby star for which we also know the parallax, we know the luminosity that corresponds to that spectral type. Nearby stars thus serve as benchmarks for more distant stars because we can assume that two stars with identical spectra have the same intrinsic luminosity.

A Few Words about the Real World

Introductory textbooks such as ours work hard to present the material in a straightforward and simplified way. In doing so, we sometimes do our students a disservice by making scientific techniques seem too clean and painless. In the real world, the techniques we have just described turn out to be messy and difficult, and often give astronomers headaches that last long into the day.

For example, the relationships we have described such as the period-luminosity relation for certain variable stars aren't exactly straight lines on a graph. The points representing many stars scatter widely when plotted, and thus, the distances derived from them also have a certain built-in scatter or uncertainty.

The distances we measure with the methods we have discussed are therefore only accurate to within a certain percentage of error—sometimes 10%, sometimes 25%, sometimes as much as 50% or more. A 25% error for a star estimated to be 10,000 light-years away means it could be anywhere from 7500 to 12,500 light-years away. This would be an unacceptable uncertainty if you were loading fuel into a spaceship for a trip to the star, but it is not a bad first figure to work with if you are an astronomer stuck on planet Earth.

Nor is the construction of H–R diagrams as easy as you might think at first. To make a good diagram, one needs to measure the characteristics and distances of many stars, which can be a time-consuming task. Since our own solar neighborhood is already well mapped, the stars astronomers most want to study to advance our knowledge are likely to be far away and faint. It may take hours of observing to obtain a single spectrum. Observers may have to spend many nights at the telescope (and many days back home working with their data) before they get their distance measurement. Fortunately, this is changing because surveys like Gaia will study billions of stars, producing public datasets that all astronomers can use.

Despite these difficulties, the tools we have been discussing allow us to measure a remarkable range of distances—parallaxes for the nearest stars, RR Lyrae variable stars; the H–R diagram for clusters of stars in our own and nearby galaxies; and cepheids out to distances of 60 million light-years. [link] describes the distance limits and overlap of each method.

Each technique described in this chapter builds on at least one other method, forming what many call the *cosmic distance ladder*. Parallaxes are the foundation of all stellar distance estimates, spectroscopic methods use nearby stars to calibrate their H–R diagrams, and RR Lyrae and cepheid distance estimates are grounded in H–R diagram distance estimates (and even in a parallax measurement to a nearby cepheid, Delta Cephei).

This chain of methods allows astronomers to push the limits when looking for even more distant stars. Recent work, for example, has used RR Lyrae stars to identify dim companion galaxies to our own Milky Way out at distances of 300,000 light-years. The H–R diagram method was recently used to identify the two most distant stars in the Galaxy: red giant stars way out in the halo of the Milky Way with distances of almost 1 million light-years.

We can combine the distances we find for stars with measurements of their composition, luminosity, and temperature—made with the techniques described in Analyzing Starlight and The Stars: A Celestial Census. Together, these make up the arsenal of information we need to trace the evolution of stars from birth to death, the subject to which we turn in the chapters that follow.

Distance Range of Celestial Measurement Methods

Method	**Distance Range**
Trigonometric parallax	4–30,000 light-years when the Gaia mission is complete
RR Lyrae stars	Out to 300,000 light-years
H–R diagram and spectroscopic distances	Out to 1,200,000 light-years
Cepheid stars	Out to 60,000,000 light-years

Three ways to find distances

Let us actually find the distance to a star using the parallax technique. The bright Southern star Beta Doradus was observed by the Fine Guidance Sensor on the Hubble Space Telescope[footnote] to have a parallax of 0.00314 ±0.000016 arc seconds, so its distance is

G. Fritz Benedict, *et al.*, The Astronomical Journal, Volume 133, Issue 4, pp. 1810-1827 (2007)

D (pc) = 1/parallax (s) = 1/(0.00314 ±0.000059 s) = 318±15 pc.

If we want to understand the *why* of stars as well as the *what*, we must add to its catalogue of extrinsic properties like distance, position in the sky, spectral type, and *apparent brightness* (b), that as seen at Earth. The first step in investigating a star's intrinsic properties is to define its *absolute brightness* or *luminosity* (L), defined as how bright it would be if it were precisely 10 pc away.

As seen in Figure 5.5, brightness falls off with the square of the distance, so the ratio of the luminosity at 10 pc, $L_{@10pc}$, to the *apparent* brightness, b, we see at its true distance will give the ratios of the squares of the star's true distance divided by 10 pc:[footnote]

The subscript is redundant but helpful to keep the first few times one encounters this idea.

$$\frac{L_{@10pc}}{b} = \frac{D^2}{(10pc)^2}$$

.

In Example 17 we presented a relationship between the ratio of brightnesses of two different stars and their *apparent magnitudes* . (The difference of the latter is proportional to the logarithm of the former.) We can use this same relation for the ratio for *absolute luminosity* (L) to the *apparent brightness* (b) of a single star in for the parallel quantities called the *absolute magnitude* (M)and *apparent magnitude* (m). That is

$$m - M = 2.5\log\left(\frac{L}{b}\right)$$

If we know the distance from its parallax, we can use distance ratio above in this expression to give the intrinsic, *absolute magnitude* (M) of any star:

$$M = m - 2.5\log\left(\frac{L}{b}\right) = m - 2.5\log\left(\left(\frac{D}{10pc}\right)^2\right) = m - 5\log\left(\frac{D}{10pc}\right)$$

,

where we have used the property of logarithms that

$$\log\left(a^p\right) = p\log\left(a\right)$$

to simplify the expression.

Beta Doradus is a Cepheid variable star whose apparent visual magnitude (m_V) (seen through a green filter) is shown fluctuating around 3.63 over a full cycle in Fig. A. Thus, its absolute magnitude (M_V) must be

$$M_V = m_V - 5\log\left(\frac{D}{10pc}\right) = 3.63 - 5\log\left(31.8 \pm 1.5\right) = 3.63 - 5\left(1.50 \pm 0.02\right)\text{=-}3.88 \pm 0.1$$

.

That is, Beta Doradus would be almost as bright as Venus if it actually were 10 pc away rather than 318.

Annie Jump Cannon was the central figure in establishing a catalogue of extrinsic spectroscopic properties of stars, a scaffold from which those who followed her might be able to garner ideas about the intrinsic properties of stars. Major progress on this road was made by her young colleague Cecilia Helena Payne (later Payne-Gaposchkin, May 10, 1900 – December 7, 1979), in 1925 the first person to earn a Ph.D. in Astronomy from Radcliff (now part of Harvard).

Her doctoral dissertation, published as “Stellar Atmospheres, A Contribution to the Observational Study of High Temperature in the Reversing Layers of Stars” is one of the most highly read doctoral dissertations of all time. Building on the ionization theory developed by Indian physicist Megh Nad Saha, she was able to show that Cannon’s classification system was a measure of stars’ temperatures. She also upended the conventional wisdom that stars’ compositions were roughly in proportion to the abundances on Earth by showing that stars contained roughly a million times more hydrogen than elements heavier than helium, a group of elements collectively referred to as “metals” by astronomers, though of course containing other sorts of heavy elements.

Modern determinations of the absolute magnitude correct for variations in a star’s proportion of such elements, called its “metallicity.” They also correct for other intrinsic effects like a Cepheid’s evolutionary stage as it passes through what is called the instability strip color of the HR diagram, and extrinsic effects of like interstellar dust that reddens and dims the light from a star. All of these effects alter m_V and hence the value we calculate for M_V. With such considerations, David G. Turner[footnote] places the absolute visual magnitude of Beta Doradus at
David G. Turner, Astrophysics and Space Science, Volume 326, Issue 2, pp 219–231, 2010.

$M_V = -3.91 \pm 0.11$.

Once one has calibrated many stars in Cannon’s classification scheme through this means, Cepheid or not, one may reverse the process and find a star’s distance from its absolute magnitude by using the other relation in Example 17,

$$\frac{L_{@10pc}}{b} = \left(100^{0.2}\right)^{m-M} = 2.512^{m-M}$$

.

This slips nicely into our distance relation to give

$$D = 10pc\sqrt{\frac{L_{@10pc}}{b}} = 10pc\sqrt{\left(100^{0.2}\right)^{m-M}} = 10pc\sqrt{2.512^{m-M}} = 10pc * 1.585^{m-M}$$

This is a helpful new rung on a distance ladder for finding distances to stars too far away for parallax measurements and is called *spectroscopic parallax*, that may be applied much further out until it become the stars become too faint to identify their spectral type.

[missing_resource: Image1.jpg]

Fig. A. Magnitudes from the fine error sensor, w(FES). Solid curves are light curves through a Visual (green) filter. [footnote]

Edward G. Schmidt and Sidney B. Parsons, The Astrophysical Journal

Supplement Series,
48:185-198, 1982.
Henrietta S. Leavitt,
Annals of Harvard
College Observatory 60,
87 (1908); Henrietta
Leavitt and Edward C.
Pickering, Harvard
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Circular 173 1-3 (1912).

Cepheid Variable stars continue play a central role in cosmology, including questions such as the age of the universe. This is due to the work of another of Annie Jump Cannon's colleagues, Henrietta Leavitt (see Figure 19.10).[footnote] It is not often that nonscientists can catch a glimpse of the creative process of scientific advancement, much less the explication of a natural law as fundamental as then one named for the more famous astronomer, Edwin Hubble, who build upon her work. The following extract is a description of what is coming to be called The Leavitt Law[footnote] in her own words at the time of its discovery:

Henrietta S. Leavitt, Annals of Harvard College Observatory 60, 87 (1908); Henrietta Leavitt and Edward C. Pickering, Harvard College Observatory Circular 173 1-3 (1912)

See for instance, "The Cepheid Period-Luminosity Relation (The Leavitt Law) at Mid-Infrared Wavelengths. II. Second-Epoch LMC Data," Barry F. Madore, Wendy L. Freedman, Jane Rigby, S. E. Persson, Laura Sturch, and Violet Mager, The Astrophysical Journal, Volume 695, Number 2 (2009); "Vetting Galactic Leavitt Law Calibrators Using Radial Velocities: On the Variability, Binarity, and Possible Parallax Error of 19 Long-Period Cepheids," Casertano, S. Riess, A. G. Melis, C. Holl, B., The Astrophysical journal Supplement series, Vol.226(2) (2016).

In H.A. 60, No. 4, attention was called to the fact that the brighter variables [2] have the longer periods, but at that time it was felt that the number was too small to warrant the drawing of general conclusions. The periods of 8 additional variables which have been determined since that time, however, conform to the same law.

TABLE I.

PERIODS OF VARIABLE STARS IN THE SMALL MAGELLANIC CLOUD.

H.	**Max.**	**Min.**	**Epoch.**	**Period**	**Res. M.**	**Res. m.**	**H.**	**Max.**	**Min.**	**Ep**
			d,	*d*,						*d*,
1505	14.8	16.1	0.02	1.25336	-0.6	-0.5	1400	14.1	14.8	4.0
1436	14.8	16.4	0.02	1.6637	-0.3	+0.1	*1355*	14.0	14.8	4.8
1446	14.8	16.4	1.38	1.7620	-0.3	+0.1	1374	13.9	15.2	6.0
1506	15.1	16.3	1.08	1.87502	+0.1	+0.1	818	13.6	14.7	4.0

H.	Max.	Min.	Epoch.	Period	Res. M.	Res. m.	H.	Max.	Min.	Ep
1413	14.7	15.6	0.35	2.17352	-0.2	-0.5	*1610*	13.4	14.6	11.
1460	14.4	15.7	0.00	2.913	-0.3	-0.1	*1365*	13.8	14.8	9.6
1422	14.7	15.9	0.6	3.501	+0.2	+0.2	*1351*	13.4	14.4	4.0
842	14.6	16.1	2.61	4.2897	+0.3	+0.6	827	13.4	14.3	11.
1425	14.3	15.3	2.8	4.547	0.0	-0.1	*822*	13.0	14.6	13.
1742	14.3	15.5	0.95	4.9866	+0.1	+0.2	823	12.2	14.1	2.9
1646	14.4	15.4	4.30	5.311	+0.3	+0.1	824	11.4	12.8	4.
1649	14.3	15.2	5.05	5.323	+0.2	-0.1	821	11.2	12.1	97.
1492	13.8	14.8	0.6	6.2926	-0.2	-0.4				

The relation is shown graphically in Figure 1, in which the abscissas are equal to the periods, expressed in days, and the ordinates are equal to the corresponding magnitudes at maxima and at minima. The two resulting curves, one for maxima and one for minima, are surprisingly smooth, and of remarkable form. In Figure 2, the abscissas are equal to the logarithms of the periods, and the ordinates to the corresponding magnitudes, as in Figure 1.

A straight line can readily be drawn among each of the two series of points corresponding to maxima and minima, thus showing that there is a simple relation between the brightness of the variables and their periods. The logarithm of the period increases by about 0.48 for each increase of one magnitude in brightness. The residuals of the maximum and minimum of each star from the lines in Figure 2 are given in the sixth and seventh columns of Table I. It is possible that the deviations from a straight line may become smaller when an absolute scale of magnitudes is used, and they may even indicate the corrections that need to be applied to the provisional scale. It should be noticed that the average range, for bright and faint [3] variables alike, is about 1.2 magnitudes. Since the variables are probably at nearly the same distance from the Earth, their periods are apparently associated with their actual emission of light, as determined by their mass, density, and surface brightness.

[missing_resource: Image2.jpg]

The faintness of the variables in the Magellanic Clouds seems to preclude the study of their spectra, with our present facilities. A number of brighter variables have similar light curves, as UY Cygni, and should repay careful study. The class of spectrum ought to be determined for as many such objects as possible. It is to be hoped, also, that the parallaxes of some variables of this type may be measured.

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$$y = s x + i,$$

where s is the slope of the line and i is the point (the *y-intercept*) where the line crosses the *y*-axis. We may read this off the upper line of her Fig. 2 as 15.6. Since we are dealing with a graph whose y-axis, the magnitudes, has values that increase downward we must take s as negative. Its size is the reciprocal of “the period [that] increases by about 0.48 for each increase of one magnitude in brightness.” Thus setting $y = mmax$ we make explicit one version of Leavitt’s Law that is laid out in English in her paper and applied to maximum brightness:

$$m\ max = -2.1\ x + 15.6\ .$$

Let us see that we have the correct form for this by applying it at a particular point, say $x = 0.8$:

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$$mV = (-2.734 \pm 0.031)\ log(P) + (15.040 \pm 0.034 + 2.734 \pm 0.031)$$

To use this we must know the period. Fig. B gives the fluctuations in the ultraviolet for Beta Doradus, plotted against the observation date.[footnote] The period can be roughly determined by counting the number of days between when the cycle climbs through a magnitude of -3.1 from one trough at about day 788 and the climb out of the next trough at about day 798, or about 10 days. Using many such cycles, the best estimate of the of period is 9.842 days.
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[missing_resource: Image3.jpg]

Table 2. OAO-2 ultraviolet light curve for ß Doradus.[footnote] James L. Hutchinson, Stephen J. Hill, Charles F. Lillie, The Astronomical Journal Volume 80, Number 12 December 1975.

$$mV = (-2.734 \pm 0.031)\ log(P) + (17.774 \pm 0.065) = 15.07 \pm 0.07$$

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M V = -2.81 log P - 1.43

One may also create a graph of this relation, Fig. C, like Leavitt’s Fig. 2.

[missing_resource: Image4.jpg]

The Leavitt Law from
Hipparcos satellite
parallax from Feast and
Catchpole·

To find the absolute magnitudes *MV* of a Cepheid with a 9.8 day period (*Log(9.8) = 0.99)*, one moves upward from *0.99* on the horizontal axis of Fig. C to the line and then moves leftward to the vertical axis, crossing at about *4.2*. This *MV* is a bit larger than Turner’s M_V = -3.91 ± 0.11 because is represents an average of many stars. Then the distance to the SMC is

$$D = 10pc * 1.585^{m-M} = 10pc * 1.585^{15.07-(-4.2)} = 10pc * 1.585^{1927} = 10pc * 7154 = 71540pc \pm 500pc$$

Instead of the exponent *m - M = 19.27*, called the *distance modulus*, Scowcroft *et al.* usinga average of 92 such stars give a value of 19.20 ± *0.03*, so we are doing pretty well. They finally correct for reddening due to intervening dust, and average several colors of light, a combined correction of 0.24 magnitudes less, to get their final distance modulus of *18.96± 0.04*., for a distance of *62 kpc ± 0.3*.

Key Concepts and Summary

Stars with identical temperatures but different pressures (and diameters) have somewhat different spectra. Spectral classification can therefore be used to estimate the luminosity class of a star as well as its temperature. As a result, a spectrum can allow us to pinpoint where the star is located on an H–R diagram and establish its luminosity. This, with the star’s apparent brightness, again yields its distance. The various distance methods can be used to check one against another and thus make a kind of distance ladder which allows us to find even larger distances.

For Further Exploration

Articles

Adams, A. “The Triumph of Hipparcos.” *Astronomy* (December 1997): 60. Brief introduction.

Dambeck, T. “Gaia’s Mission to the Milky Way.” *Sky & Telescope* (March 2008): 36–39. An introduction to the mission to measure distances and positions of stars with unprecedented accuracy.

Hirshfeld, A. “The Absolute Magnitude of Stars.” *Sky & Telescope* (September 1994): 35. Good review of how we measure luminosity, with charts.

Hirshfeld, A. “The Race to Measure the Cosmos.” *Sky & Telescope* (November 2001): 38. On parallax.

Trefil, J. Puzzling Out Parallax.” *Astronomy* (September 1998): 46. On the concept and history of parallax.

Turon, C. “Measuring the Universe.” *Sky & Telescope* (July 1997): 28. On the Hipparcos mission and its results.

Zimmerman, R. “Polaris: The Code-Blue Star.” *Astronomy* (March 1995): 45. On the famous cepheid variable and how it is changing.

Websites

ABCs of Distance: http://www.astro.ucla.edu/~wright/distance.htm. Astronomer Ned Wright (UCLA) gives a concise primer on many different methods of obtaining distances. This site is at a higher level than our textbook, but is an excellent review for those with some background in astronomy.

American Association of Variable Star Observers (AAVSO): https://www.aavso.org/. This organization of amateur astronomers helps to keep track of variable stars; its site has some background material, observing instructions, and links.

Friedrich Wilhelm Bessel: http://messier.seds.org/xtra/Bios/bessel.html. A brief site about the first person to detect stellar parallax, with references and links.

Gaia: http://sci.esa.int/gaia/. News from the Gaia mission, including images and a blog of the latest findings.

Hipparchos: http://sci.esa.int/hipparcos/. Background, results, catalogs of data, and educational resources from the Hipparchos mission to observe parallaxes from space. Some sections are technical, but others are accessible to students.

John Goodricke: The Deaf Astronomer: http://www.bbc.com/news/magazine-20725639. A biographical article from the BBC.

Women in Astronomy: http://www.astrosociety.org/education/astronomy-resource-guides/women-in-astronomy-an-introductory-resource-guide/. More about Henrietta Leavitt's and other women's contributions to astronomy and the obstacles they faced.

Videos

Gaia's Mission: Solving the Celestial Puzzle: https://www.youtube.com/watch?v=oGri4YNggoc. Describes the Gaia mission and what scientists hope to learn, from Cambridge University (19:58).

Hipparcos: Route Map to the Stars: https://www.youtube.com/watch?v=4d8a75fs7KI. This ESA video describes the mission to measure parallax and its results (14:32)

How Big Is the Universe: https://www.youtube.com/watch?v=K_xZuopg4Sk. Astronomer Pete Edwards from the British Institute of Physics discusses the size of the universe and gives a step-by-step introduction to the concepts of distances (6:22)

Search for Miss Leavitt: http://perimeterinstitute.ca/videos/search-miss-leavitt., Video of talk by George Johnson on his search for Miss Leavitt (55:09).

Women in Astronomy: http://www.youtube.com/watch?v=5vMR7su4fi8. Emily Rice (CUNY) gives a talk on the contributions of women to astronomy, with many historical and contemporary examples, and an analysis of modern trends (52:54).

Collaborative Group Activities

A. In this chapter, we explain the various measurements that have been used to establish the size of a standard meter. Your group should discuss why we have changed the definitions of our standard unit of measurement in science from time to time. What factors in our modern society contribute to the growth of technology? Does technology "drive" science, or does science "drive" technology? Or do you think the two are so intertwined that it's impossible to say which is the driver?
B. Cepheids are scattered throughout our own Milky Way Galaxy, but the period-luminosity relation was discovered from observations of the Magellanic Clouds, a satellite galaxy now known to be about 160,000 light-years away. What reasons can you give to explain why the relation was not discovered from

observations of cepheids in our own Galaxy? Would your answer change if there were a small cluster in our own Galaxy that contained 20 cepheids? Why or why not?

C. You want to write a proposal to use the Hubble Space Telescope to look for the brightest cepheids in galaxy M100 and estimate their luminosities. What observations would you need to make? Make a list of all the reasons such observations are harder than it first might appear.
D. Why does your group think so many different ways of naming stars developed through history? (Think back to the days before everyone connected online.) Are there other fields where things are named confusingly and arbitrarily? How do stars differ from other phenomena that science and other professions tend to catalog?
E. Although cepheids and RR Lyrae variable stars tend to change their brightness pretty regularly (while they are in that stage of their lives), some variable stars are unpredictable or change their their behavior even during the course of a single human lifetime. Amateur astronomers all over the world follow such variable stars patiently and persistently, sending their nightly observations to huge databases that are being kept on the behavior of many thousands of stars. None of the hobbyists who do this get paid for making such painstaking observations. Have your group discuss why they do it. Would you ever consider a hobby that involves so much work, long into the night, often on work nights? If observing variable stars doesn’t pique your interest, is there something you think you could do as a volunteer after college that does excite you? Why?
F. In [link], the highest concentration of stars occurs in the middle of the main sequence. Can your group give reasons why this might be so? Why are there fewer very hot stars and fewer very cool stars on this diagram?
G. In this chapter, we discuss two astronomers who were differently abled than their colleagues. John Goodricke could neither hear nor speak, and Henrietta Leavitt struggled with hearing impairment for all of her adult life. Yet they each made fundamental contributions to our understanding of the universe. Does your group know people who are handling a disability? What obstacles would people with different disabilities face in trying to do astronomy and what could be done to ease their way? For a set of resources in this area, see http://astronomerswithoutborders.org/gam2013/programs/1319-people-with-disabilities-astronomy-resources.html.

Review Questions

Exercise:

Problem:

Explain how parallax measurements can be used to determine distances to stars. Why can we not make accurate measurements of parallax beyond a certain distance?

Exercise:

Problem:

Suppose you have discovered a new cepheid variable star. What steps would you take to determine its distance?

Exercise:

Problem: Explain how you would use the spectrum of a star to estimate its distance.

Exercise:

Problem: Which method would you use to obtain the distance to each of the following?

A. An asteroid crossing Earth’s orbit
B. A star astronomers believe to be no more than 50 light-years from the Sun
C. A tight group of stars in the Milky Way Galaxy that includes a significant number of variable stars
D. A star that is not variable but for which you can obtain a clearly defined spectrum

Exercise:

Problem:

What are the luminosity class and spectral type of a star with an effective temperature of 5000 K and a luminosity of 100 L_{Sun}?

Thought Questions

Exercise:

Problem:

The meter was redefined as a reference to Earth, then to krypton, and finally to the speed of light. Why do you think the reference point for a meter continued to change?

Exercise:

Problem:

While a meter is the fundamental unit of length, most distances traveled by humans are measured in miles or kilometers. Why do you think this is?

Exercise:

Problem:

Most distances in the Galaxy are measured in light-years instead of meters. Why do you think this is the case?

Exercise:

Problem:

The AU is defined as the *average* distance between Earth and the Sun, not the distance between Earth and the Sun. Why does this need to be the case?

Exercise:

Problem:

What would be the advantage of making parallax measurements from Pluto rather than from Earth? Would there be a disadvantage?

Exercise:

Problem:

Parallaxes are measured in fractions of an arcsecond. One arcsecond equals 1/60 arcmin; an arcminute is, in turn, 1/60th of a degree (°). To get some idea of how big 1° is, go outside at night and find the Big Dipper. The two pointer stars at the ends of the bowl are 5.5° apart. The two stars across the top of the bowl are 10° apart. (Ten degrees is also about the width of your fist when held at arm's length and projected against the sky.) Mizar, the second star from the end of the Big Dipper's handle, appears double. The fainter star, Alcor, is about 12 arcmin from Mizar. For comparison, the diameter of the full moon is about 30 arcmin. The belt of Orion is about 3° long. Keeping all this in mind, why did it take until 1838 to make parallax measurements for even the nearest stars?

Exercise:

Problem:

For centuries, astronomers wondered whether comets were true celestial objects, like the planets and stars, or a phenomenon that occurred in the atmosphere of Earth. Describe an experiment to determine which of these two possibilities is correct.

Exercise:

Problem:

The Sun is much closer to Earth than are the nearest stars, yet it is not possible to measure accurately the diurnal parallax of the Sun relative to the stars by measuring its position relative to background objects in the sky directly. Explain why.

Exercise:

Problem:

Parallaxes of stars are sometimes measured relative to the positions of galaxies or distant objects called quasars. Why is this a good technique?

Exercise:

Problem:

Estimating the luminosity class of an M star is much more important than measuring it for an O star if you are determining the distance to that star. Why is that the case?

Exercise:

Problem:

[link] is the light curve for the prototype cepheid variable Delta Cephei. How does the luminosity of this star compare with that of the Sun?

Exercise:

Problem:

Which of the following can you determine about a star without knowing its distance, and which can you not determine: radial velocity, temperature, apparent brightness, or luminosity? Explain.

Exercise:

Problem:

A G2 star has a luminosity 100 times that of the Sun. What kind of star is it? How does its radius compare with that of the Sun?

Exercise:

Problem: A star has a temperature of 10,000 K and a luminosity of $10^{-2}\ L_{\text{Sun}}$. What kind of star is it?

Exercise:

Problem:

What is the advantage of measuring a parallax distance to a star as compared to our other distance measuring methods?

Exercise:

Problem:

What is the disadvantage of the parallax method, especially for studying distant parts of the Galaxy?

Exercise:

Problem:

Luhman 16 and WISE 0720 are brown dwarfs, also known as failed stars, and are some of the new closest neighbors to Earth, but were only discovered in the last decade. Why do you think they took so long to be discovered?

Exercise:

Problem:

Most stars close to the Sun are red dwarfs. What does this tell us about the average star formation event in our Galaxy?

Exercise:

Problem:

Why would it be easier to measure the characteristics of intrinsically less luminous cepheids than more luminous ones?

Exercise:

Problem:

When Henrietta Leavitt discovered the period-luminosity relationship, she used cepheid stars that were all located in the Large Magellanic Cloud. Why did she need to use stars in another galaxy and not cepheids located in the Milky Way?

Figuring for Yourself

Exercise:

Problem:

A radar astronomer who is new at the job claims she beamed radio waves to Jupiter and received an echo exactly 48 min later. Should you believe her? Why or why not?

Exercise:

Problem:

The New Horizons probe flew past Pluto in July 2015. At the time, Pluto was about 32 AU from Earth. How long did it take for communication from the probe to reach Earth, given that the speed of light in km/hr is 1.08×10^9?

Exercise:

Problem:

Estimate the maximum and minimum time it takes a radar signal to make the round trip between Earth and Venus, which has a semimajor axis of 0.72 AU.

Exercise:

Problem:

The Apollo program (not the lunar missions with astronauts) being conducted at the Apache Point Observatory uses a 3.5-m telescope to direct lasers at retro-reflectors left on the Moon by the Apollo astronauts. If the Moon is 384,472 km away, approximately how long do the operators need to wait to see the laser light return to Earth?

Exercise:

Problem:

In 1974, the Arecibo Radio telescope in Puerto Rico was used to transmit a signal to M13, a star cluster about 25,000 light-years away. How long will it take the message to reach M13, and how far has the message travelled so far (in light-years)?

Exercise:

Problem:

Demonstrate that 1 pc equals 3.09×10^{13} km and that it also equals 3.26 light-years. Show your calculations.

Exercise:

Problem:

The best parallaxes obtained with Hipparcos have an accuracy of 0.001 arcsec. If you want to measure the distance to a star with an accuracy of 10%, its parallax must be 10 times larger than the typical error. How far away can you obtain a distance that is accurate to 10% with Hipparcos data? The disk of our Galaxy is 100,000 light-years in diameter. What fraction of the diameter of the Galaxy's disk is the distance for which we can measure accurate parallaxes?

Exercise:

Problem:

Astronomers are always making comparisons between measurements in astronomy and something that might be more familiar. For example, the Hipparcos web pages tell us that the measurement accuracy of 0.001 arcsec is equivalent to the angle made by a golf ball viewed from across the Atlantic Ocean, or to the angle made by the height of a person on the Moon as viewed from Earth, or to the length of growth of a human hair in 10 sec as seen from 10 meters away. Use the ideas in [link] to verify one of the first two comparisons.

Exercise:

Problem:

Gaia will have greatly improved precision over the measurements of Hipparcos. The average uncertainty for most *Gaia* parallaxes will be about 50 microarcsec, or 0.00005 arcsec. How many times better than Hipparcos (see [link]) is this precision?

Exercise:

Problem:

Using the same techniques as used in [link], how far away can *Gaia* be used to measure distances with an uncertainty of 10%? What fraction of the Galactic disk does this correspond to?

Exercise:

Problem:

The human eye is capable of an angular resolution of about one arcminute, and the average distance between eyes is approximately 2 in. If you blinked and saw something move about one arcmin across, how far away from you is it? (Hint: You can use the setup in [link] as a guide.)

Exercise:

Problem:

How much better is the resolution of the *Gaia* spacecraft compared to the human eye (which can resolve about 1 arcmin)?

Exercise:

Problem:

The most recently discovered system close to Earth is a pair of brown dwarfs known as Luhman 16. It has a distance of 6.5 light-years. How many parsecs is this?

Exercise:

Problem: What would the parallax of Luhman 16 (see [link]) be as measured from Earth?

Exercise:

Problem:

The New Horizons probe that passed by Pluto during July 2015 is one of the fastest spacecraft ever assembled. It was moving at about 14 km/s when it went by Pluto. If it maintained this speed, how long would it take New Horizons to reach the nearest star, Proxima Centauri, which is about 4.3 light-years away? (Note: It isn't headed in that direction, but you can pretend that it is.)

Exercise:

Problem:

What physical properties are different for an M giant with a luminosity of 1000 L_{Sun} and an M dwarf with a luminosity of 0.5 L_{Sun}? What physical properties are the same?

Exercise:

Problem:

The star TRAPPIST-1 has been found to have seven terrestrial planets within its habitable zone. Its parallax is 0.08258 ± 0.000258 arc seconds. It has apparent magnitude $m_V = 18.80$ and absolute magnitude $M_V = 18.4 \pm 0.1$. Use both methods at your disposal to find its distance. How do they compare?

Exercise:

Problem:

The visual light curve[footnote] is shown for the Cepheid Variable star HV 12198 in the Large Magellanic Cloud (LMC). It has apparent magnitude $m_V = 15.77$. Measure its period. Use the Leavitt Law (using the graph or Feast and Catchpole's equation[10]) to get its absolute magnitude M_V. Correct the distance modulus $m_V - M_V$ for intervening dust and by subtracting the same 0.24 magnitudes that Scowcroft *et al.* did for the SMC.[7>] Find the distance to the LMC.

Data from "CCD light curves for seven Cepheid variables in the LMC cluster NGC 1866," Walker, A. R., Journal: Monthly Notices of the Royal Astronomical Society (ISSN 0035-8711), vol. 225, p. 627-641, (1987).

`[missing_resource: Image1.jpg]`

Glossary

luminosity class	a classification of a star according to its luminosity within a given spectral class; our Sun, a G2V star, has luminosity class V, for example

two new distance exercises for Chaper 19
two new distance exercises for Chaper 19 of Astronomy

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Feast MW, Catchpole RM. 1997. MNRAS 286:L1
Victoria Scowcroft, Wendy L. Freedman, Barry F. Madore, Andy Monson, S. E. Persson, Jeff Rich, Mark Seibert, and Jane R. Rigby, The Astrophysical Journal, Volume 816, Number 2 (2016). In their notation $mV = a_V\ (log(P)\ -1.0) + b_V$ with $a_V = (-2.734 \pm 0.031)$ and $b_V = (15.040 \pm 0.034)$.